Solitonic excitations in nuclear reactions and their counterparts in ultracold Fermi gases

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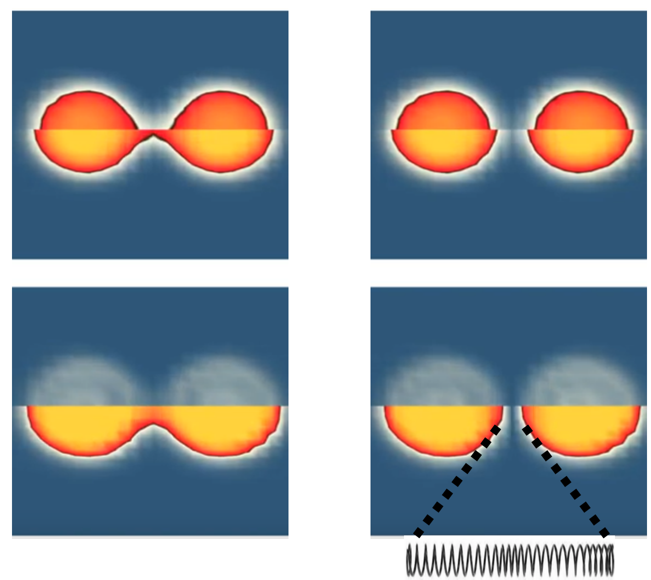

TDDFT simulation

https://www.youtube.com/watch?v=k5bNybqvRRc

TDDFT simulation
https://www.youtube.com/watch?v=k5bNybqvRRc

The results (may) depend on:

1) Energy Density Functional (we use Skyrme type functional)

Featured in Physics Free to Read

Inhomogeneous Electron Gas

P. Hohenberg and W. Kohn
Phys. Rev. **136**, B864 – Published 9 November 1964

Physics

Article | References | Citing Articles (37,261) | PDF | Export Citation

Note: In case of self-bound systems one needs to replace "laboratory density" with the "intrinsic density" in the HK theorem.

J. Engel, Phys. Rev. C 75, 014306 (2006);
N. Barnea Phys. Rev. C 76, 067302 (2007);
J. Messud, M. Bender, E. Suraud,
Phys. Rev. C 80, 054314 (2009);

- DFT is in principle exact theory
  Hohenberg-Kohn theorem (1964) implies that
- ... solving Schrödinger equation ↔ minimization of the energy density E[ρ]...
- ... however no mathematical recipe how to construct E[ρ].
- In practice we postulate the functional form!

TDDFT simulation
https://www.youtube.com/watch?v=k5bNybqvRRc

The results (may) depend on:

1) Energy Density Functional (we use Skyrme type functional)
2) Broken symmetries: superfluidity/superconductivity → particle number symmetry is broken

Density-Functional Theory for Superconductors
L. N. Oliveira, E. K. U. Gross, and W. Kohn
Phys. Rev. Lett. **60**, 2430 – Published 6 June 1988

- DFT has been extended to superfluid/superconducting systems...
- ... energy density functional $E[\rho,\nu]$...
- ... where ν is anomalous density ↔ order parameter (quantity well defined in thermodynamic limit)

TDDFT simulation
https://www.youtube.com/watch?v=k5bNybqvRRc

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3) Adiabatic approximation: $H\varphi_n = E_n\varphi_n \rightarrow i\hbar\frac{\partial\varphi_n}{\partial t} = H\varphi_n$

Density-Functional Theory for Time-Dependent Systems

Erich Runge and E. K. U. Gross
Phys. Rev. Lett. **52**, 997 – Published 19 March 1984

Time-Dependent Density-Functional Theory for Superconductors

O. -J. Wacker, R. Kümmel, and E. K. U. Gross
Phys. Rev. Lett. **73**, 2915 – Published 21 November 1994

- For time-dependent case the “exact” functional is in general different from the one that is used in static calculations...

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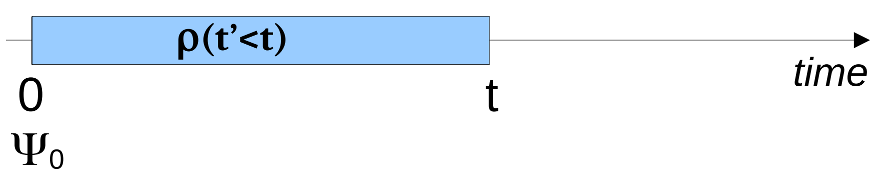
$$E(t) = E[\Psi(t=0), \rho(r, t' \leqslant t)]$$



$$E(t) = \int_V d\boldsymbol{r}\, \mathcal{E}[\rho(\boldsymbol{r}, t), \ldots]$$

Adiabatic approximation

In general integro-differential equations ← $E(t) = \int_0^t dt' \int_V d\boldsymbol{r}\, \mathcal{E}[\Psi_0, \rho(\boldsymbol{r}, t'), \ldots]$

- For time-dependent case the "exact" functional is in general different from the one that is used in static calculations...
- ... but if the evolution is slow (adiabatic), the system follows instantaneous ground state → use the functional taken from static considerations.

TDDFT simulation
https://www.youtube.com/watch?v=k5bNybqvRRc

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4) Practical/Numerical implementation (discretization scheme, regularization, ...)

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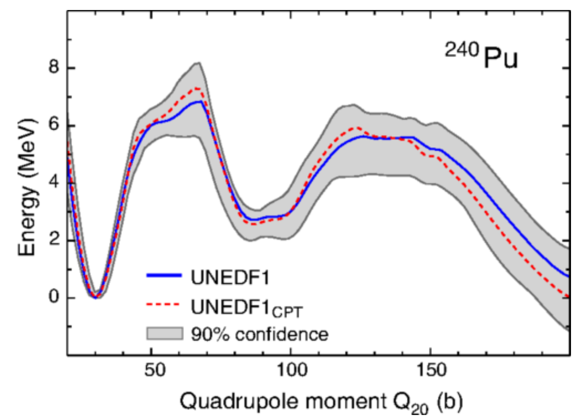
4) Practical/Numerical implementation (discretization scheme, regularization, ...)

- Uncertainty quantification needed...
- ... presently no well defined methodology how to do it in case of time-dependent phenomena.
- Experiments provide info only about final states, no detailed info about time-evolution.

Fig. From: J. D. McDonnell, N. Schunck, D. Higdon, J. Sarich, S. M. Wild, and W. Nazarewicz, Phys. Rev. Lett. 114, 122501 (2015)



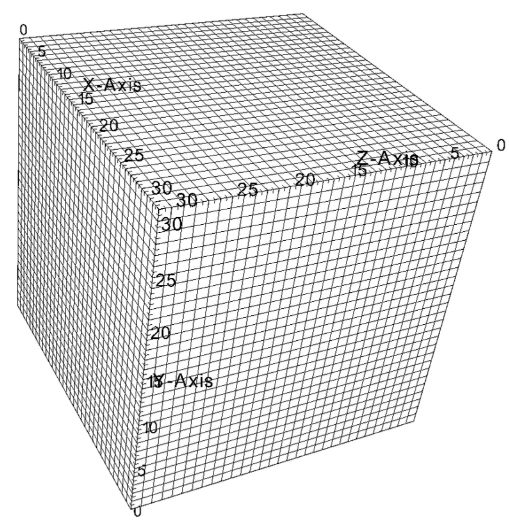
The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ u_{n,\downarrow}(\boldsymbol{r},t) \\ v_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\boldsymbol{r},t) & h_{\uparrow,\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow,\uparrow}(\boldsymbol{r},t) & h_{\downarrow,\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow,\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow,\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow,\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow,\downarrow}(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ u_{n,\downarrow}(\boldsymbol{r},t) \\ v_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

where *h* and Δ depends on "densities" (their explicit form depends on the functional):

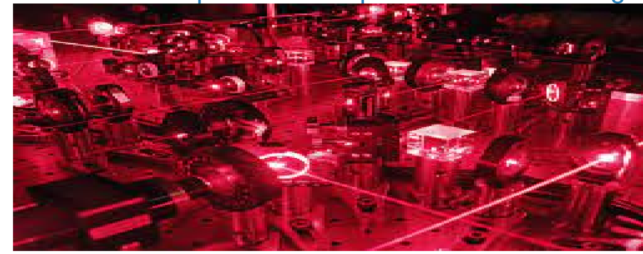
$$\rho_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \nu(\boldsymbol{r},t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \text{etc.}$$

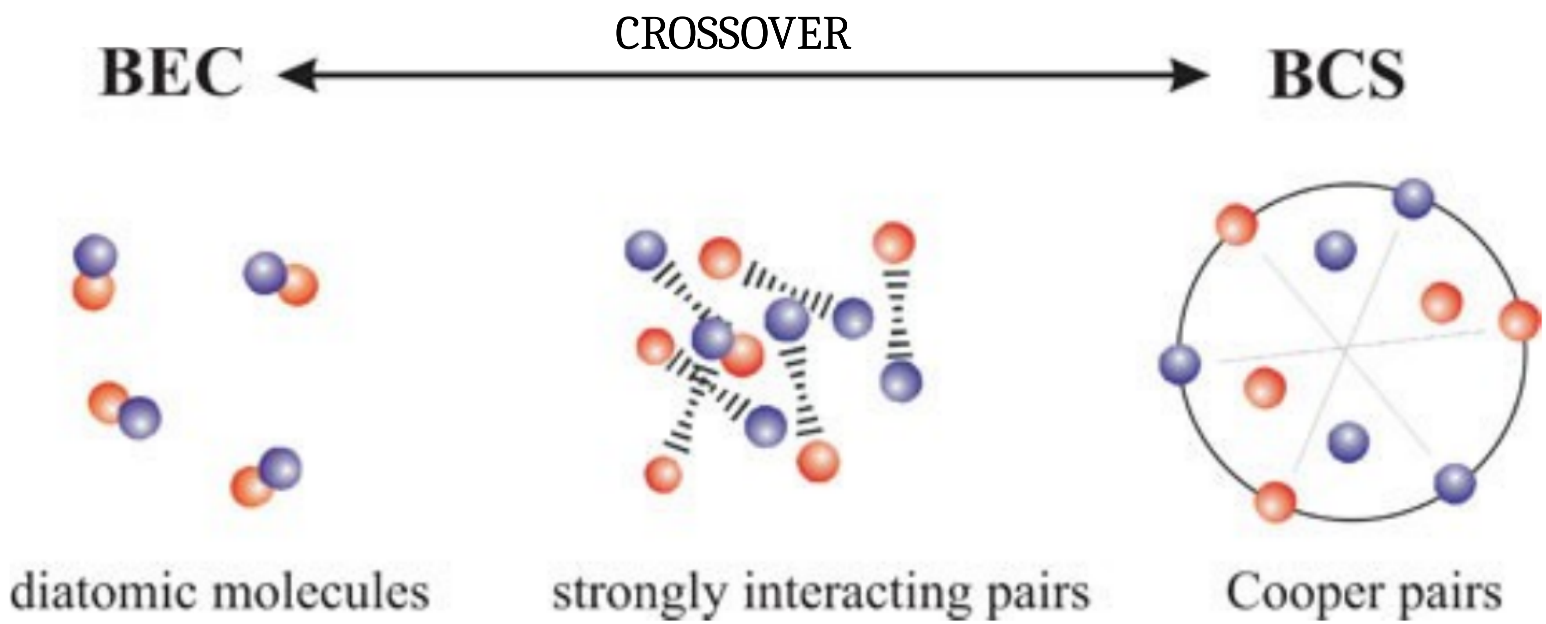
- Mathematically: large set of nonlinear coupled 3D PDEs;
  Number of PDEs is of the order of the number of spatial lattice points, typically 10^5-10^6.
- We use coordinate space solvers (system is placed on a 3D spatial lattice of size $N_x \times N_y \times N_z$)
- Codes are publicly accessible:
  Nuclear → The LISE Package: Comput. Phys. Commun. 269, 108130 (2021).
  Cold atoms → W-SLDA Toolkit, https://wslda.fizyka.pw.edu.pl

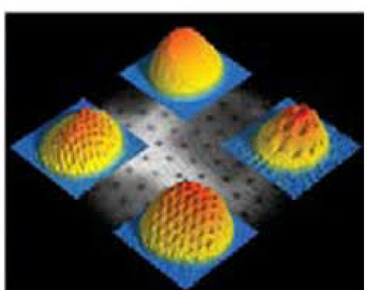


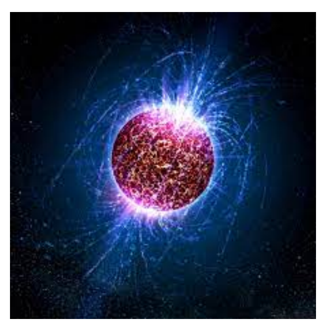
- Ultracold atomic systems offer alternative possibility to test predictive power of TDDFT.
- The (bare) interaction is simple $V(r-r')=g\delta(r-r')$...
- ... but the interaction strength g can be tuned at will!

Photo from: http://www.lkb.upmc.fr/ultracoldfermigases/









Modeling nuclear vs ultracold systems

1) Energy Density Functional

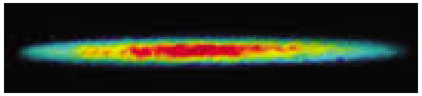
The energy functional for cold atoms is much simpler than for nuclear cases *(no gradient terms, no spin-orbit, …)*

[We use SLDA functional, originally derived by A. Bulgac, Phys. Rev. A 76, 040502(R)]

2) Broken symmetries: superfluidity/superconductivity

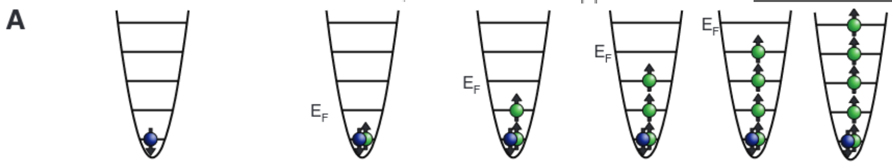
→ *can be easily justified since number of atoms in the cloud is 10^5-10^6*

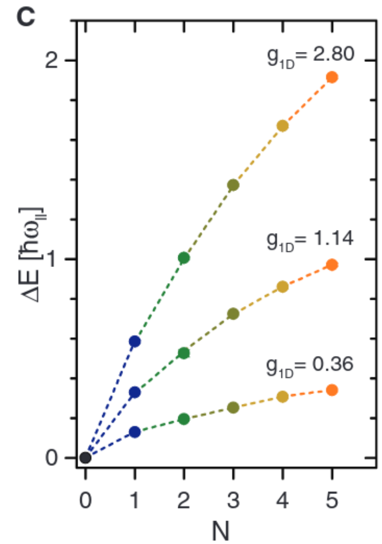
… but systems with “small” number of atoms are also accessible



Remark: transition from few-body to many-physics is also active area of researches with ultracold atoms...

SCIENCE • 25 Oct 2013 • Vol 342, Issue 6157 • pp. 457-460 • DOI: 10.1126/science.1240516





From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz,[1,2]*† G. Zürn,[1,2]† S. Murmann,[1,2] I. Brouzos,[3] T. Lompe,[1,2,4] S. Jochim[1,2,4]

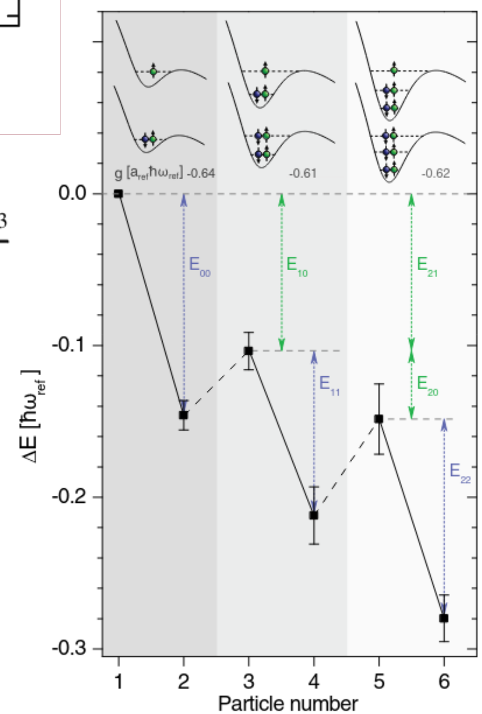
Knowing when a physical system has reached sufficient size for its macroscopic properties to be well described by many-body theory is difficult. We investigated the crossover from few- to many-body physics by studying quasi–one-dimensional systems of ultracold atoms consisting of a single impurity interacting with an increasing number of identical fermions.

PRL **111,** 175302 (2013) PHYSICAL REVIEW LETTERS week ending 25 OCTOBER 2013

Pairing in Few-Fermion Systems with Attractive Interactions

G. Zürn,[1,2,*] A. N. Wenz,[1,2] S. Murmann,[1,2] A. Bergschneider,[1,2] T. Lompe,[1,2,3] and S. Jochim[1,2,3]

We study quasi-one-dimensional few-particle systems consisting of one to six ultracold fermionic atoms in two different spin states with attractive interactions. We probe the system by deforming the trapping potential and by observing the tunneling of particles out of the trap. For even particle numbers, we observe a tunneling behavior that deviates from uncorrelated single-particle tunneling indicating the existence of pair correlations in the system. From the tunneling time scales, we infer the differences in interaction energies of systems with different number of particles, which show a strong odd-even effect, similar to the one observed for neutron separation experiments in nuclei.



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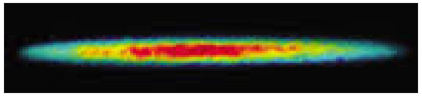
2) Broken symmetries: superfluidity/superconductivity

→ *can be easily justified since number of atoms in the cloud is 10^5-10^6*



… but systems with "small" number of atoms are also accessible

3) Adiabatic approximation: $H\varphi_n = E_n\varphi_n \rightarrow i\hbar\frac{\partial\varphi_n}{\partial t} = H\varphi_n$

No change

4) Practical/Numerical implementation (discretization scheme, regularization, ….)

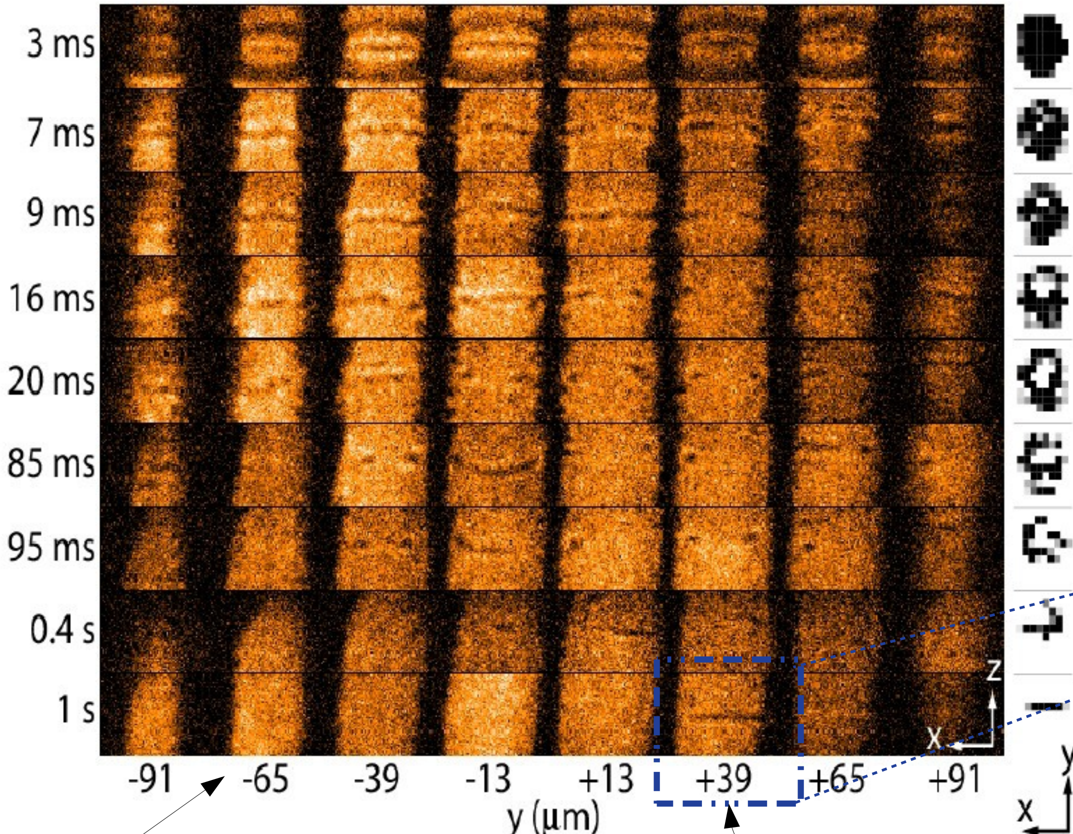
No change

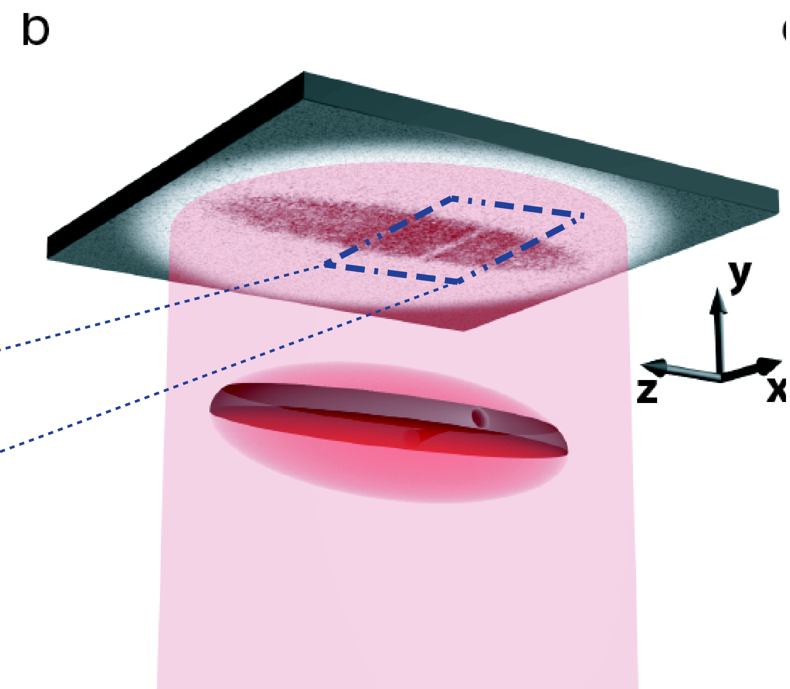
(a)

experiment

Phys. Rev. Lett. 116, 045304 (2016)

Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay of a dark soliton into a vortex line





Strongly interacting Fermi gas

Note:

here we observe directly time evolution of

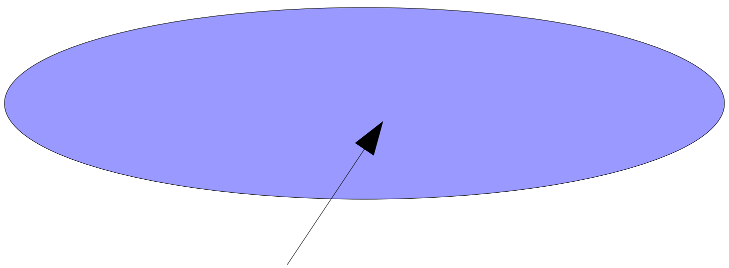
density ρ(r,t)

for quantum system

Superfluidity: U(1) order parameter

$$\Delta(\boldsymbol{r}) = |\Delta(\boldsymbol{r})|e^{i\theta(\boldsymbol{r})}$$

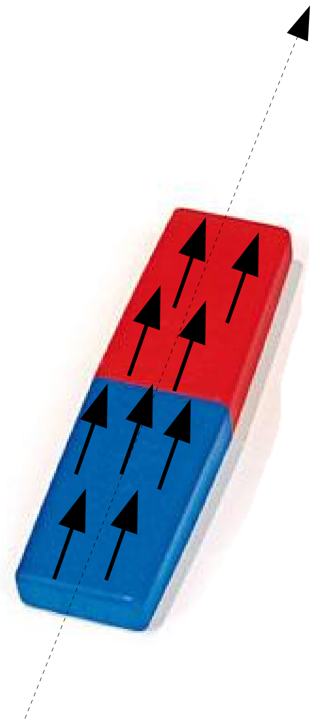
Order parameter = “pairing” Δ



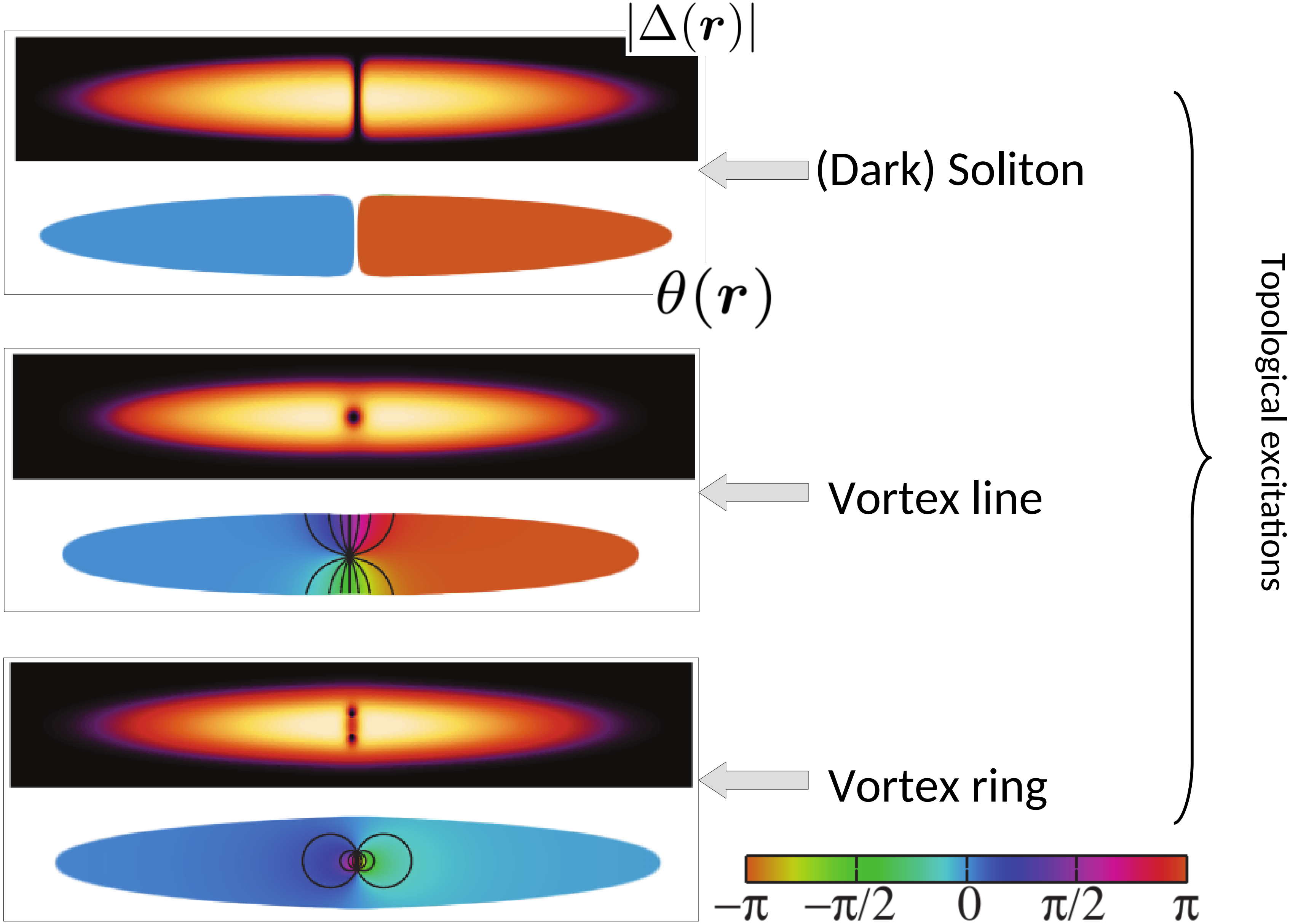
In a ground state the phase
is uniform across the system…

… and since it is closely connected
with phase of the wave-function
it can be absorbed by
normalization factor

$$\Delta(\boldsymbol{r}) \rightarrow |\Delta(\boldsymbol{r})|$$



Excited states



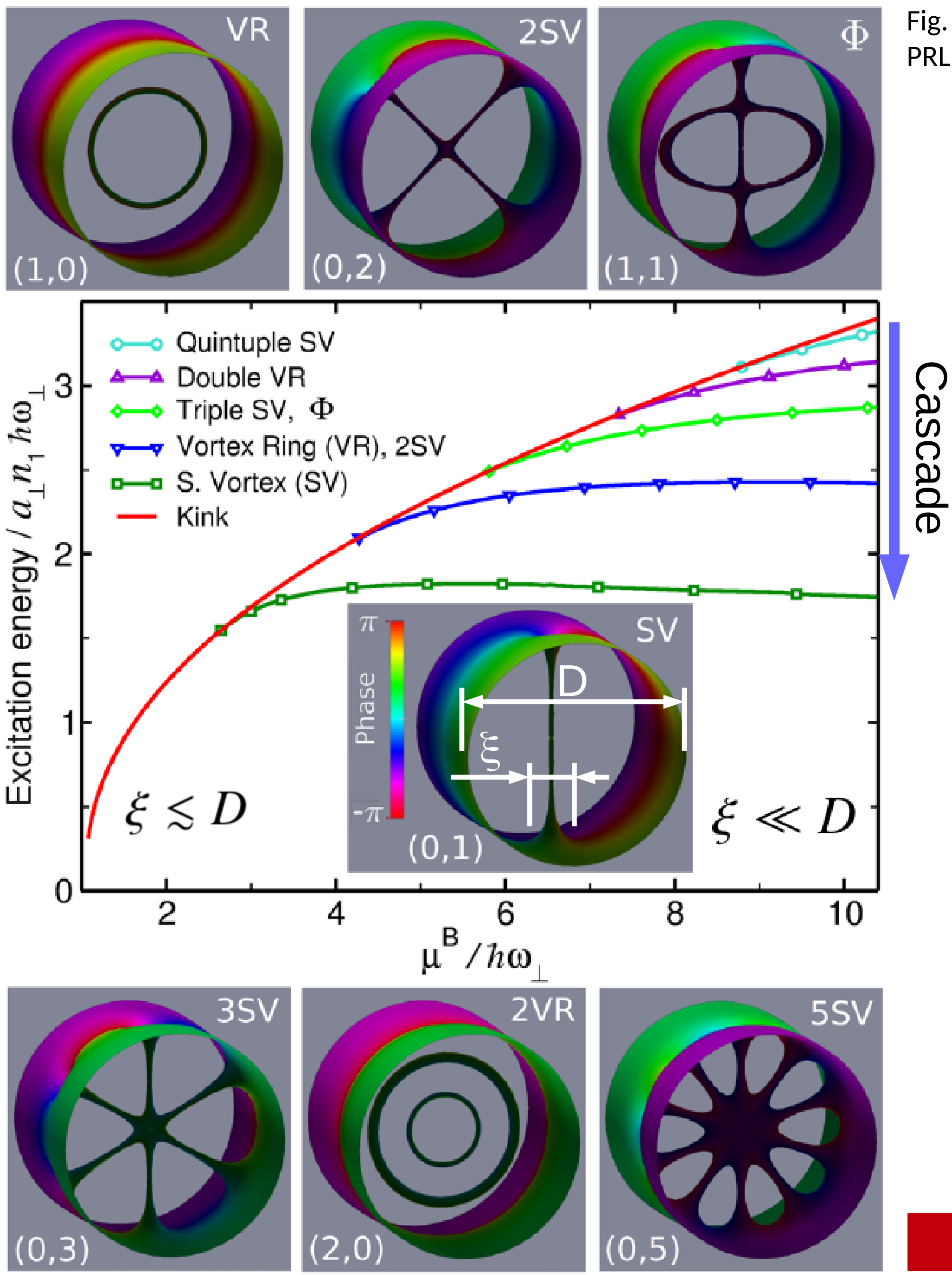


Fig. from: A. Munoz Mateo and J. Brand, PRL 113, 255302 (2014)

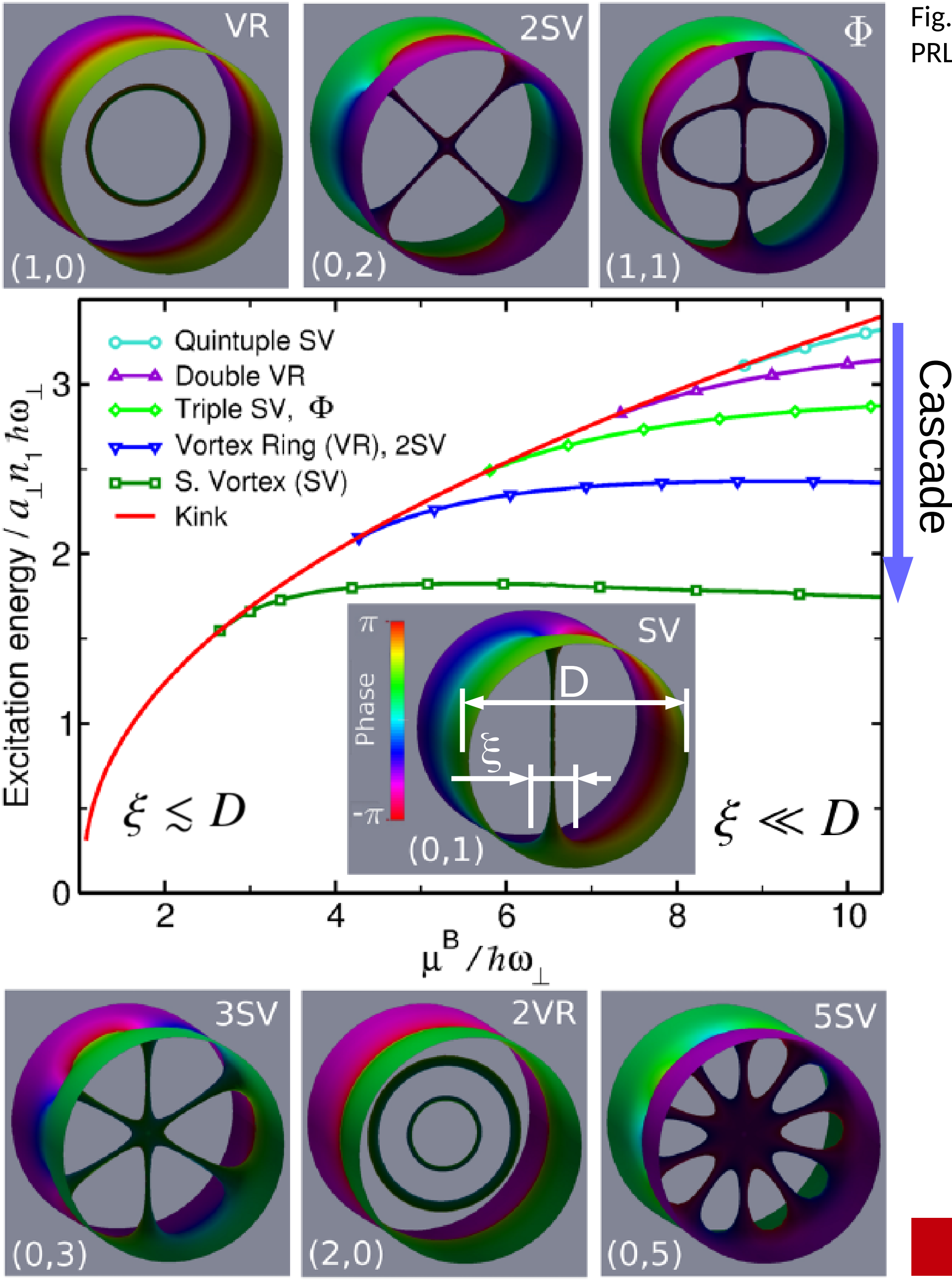


Fig. from: A. Munoz Mateo and J. Brand, PRL 113, 255302 (2014)

Coherence length: $\xi = \dfrac{\hbar v_F}{\pi \Delta}$

Cold atoms at unitarity: $\xi \sim n^{-1/3} \ll L$

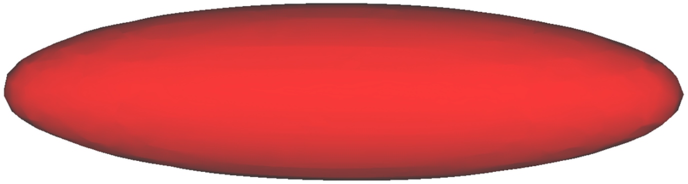
Nuclear systems (n=0.08fm-3):

$\xi \approx 17\,\mathrm{fm} \approx 2R$ where R is radius of heavy nucleus

GENERAL REMARKS:

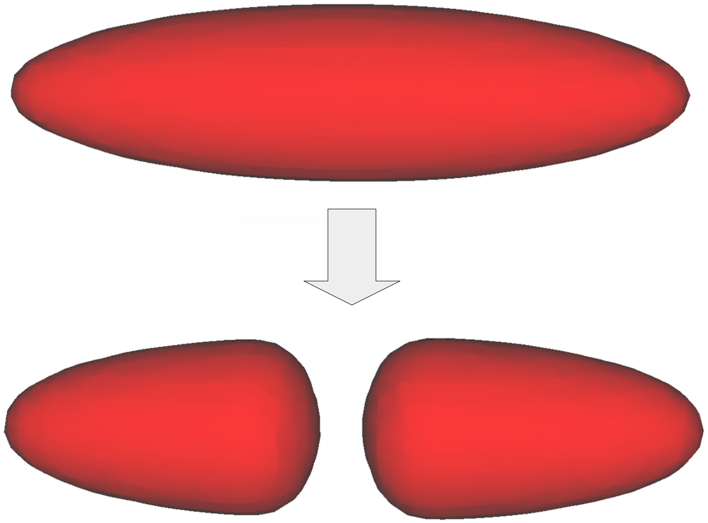
- Δ(r) should be treated as dynamical field...
- the Δ(r) field introduces new excitation modes to the system (enormous number of new modes)
  - fluctuations (waves) of $|\Delta$(r)$|$...
  - fluctuations (waves) of θ(r)...
  - Solitonic excitations..

How to excite “solitonic” modes?



1. Start with a cloud in ground state...

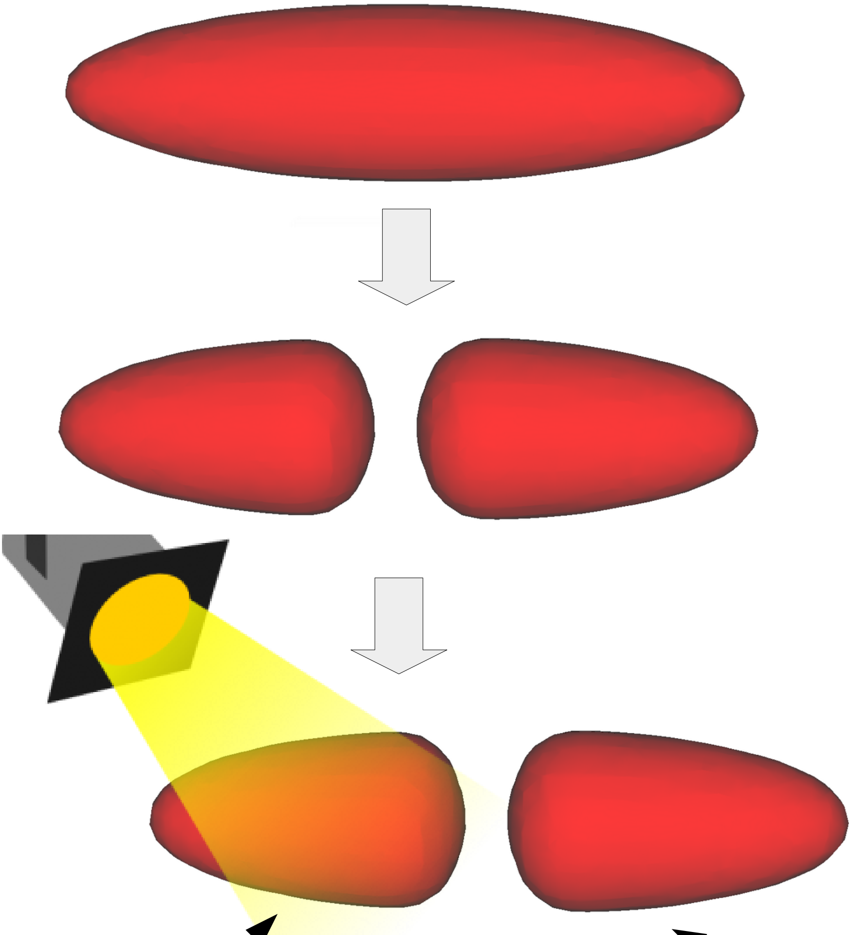
How to excite “solitonic” modes?



1. Start with a cloud in ground state...

2. Split into two fragments ...

How to excite “solitonic” modes?



1. Start with a cloud in ground state...

2. Split into two fragments ...

3. Add constant potential U to one part ...

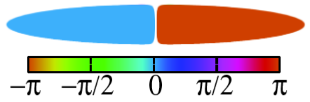
$$\Psi_1 \sim e^{-i(E+U)t/\hbar}\psi \qquad \Psi_2 \sim e^{-iEt/\hbar}\psi$$

$$\theta_1 - \theta_2 = Ut/\hbar$$

Note: uniquely defined quantity

MIT experiments:
Nature 499, 426 (2013);
Phys. Rev. Lett. 113, 065301(2014);
Phys. Rev. Lett. 116, 045304 (2016);

- ^{6}Li atoms near a Feshbach resonance (N≈10^6) cooled in harmonic trap
- Step potential used to imprint a soliton (evolve to π phase shift)



- Let system evolve...
- Take picture (subtle imaging with tomography)

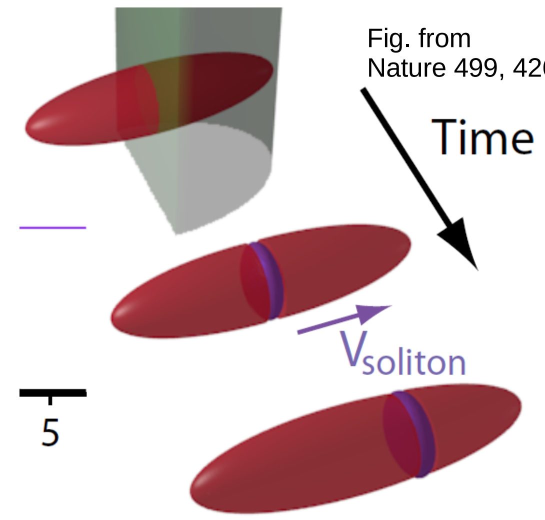


Fig. from
Nature 499, 426 (2013)

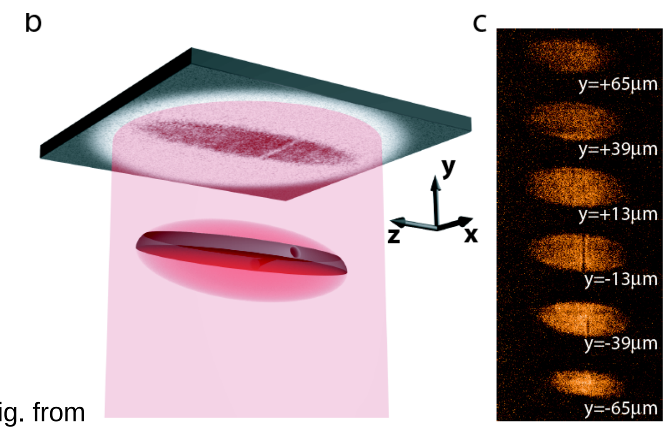
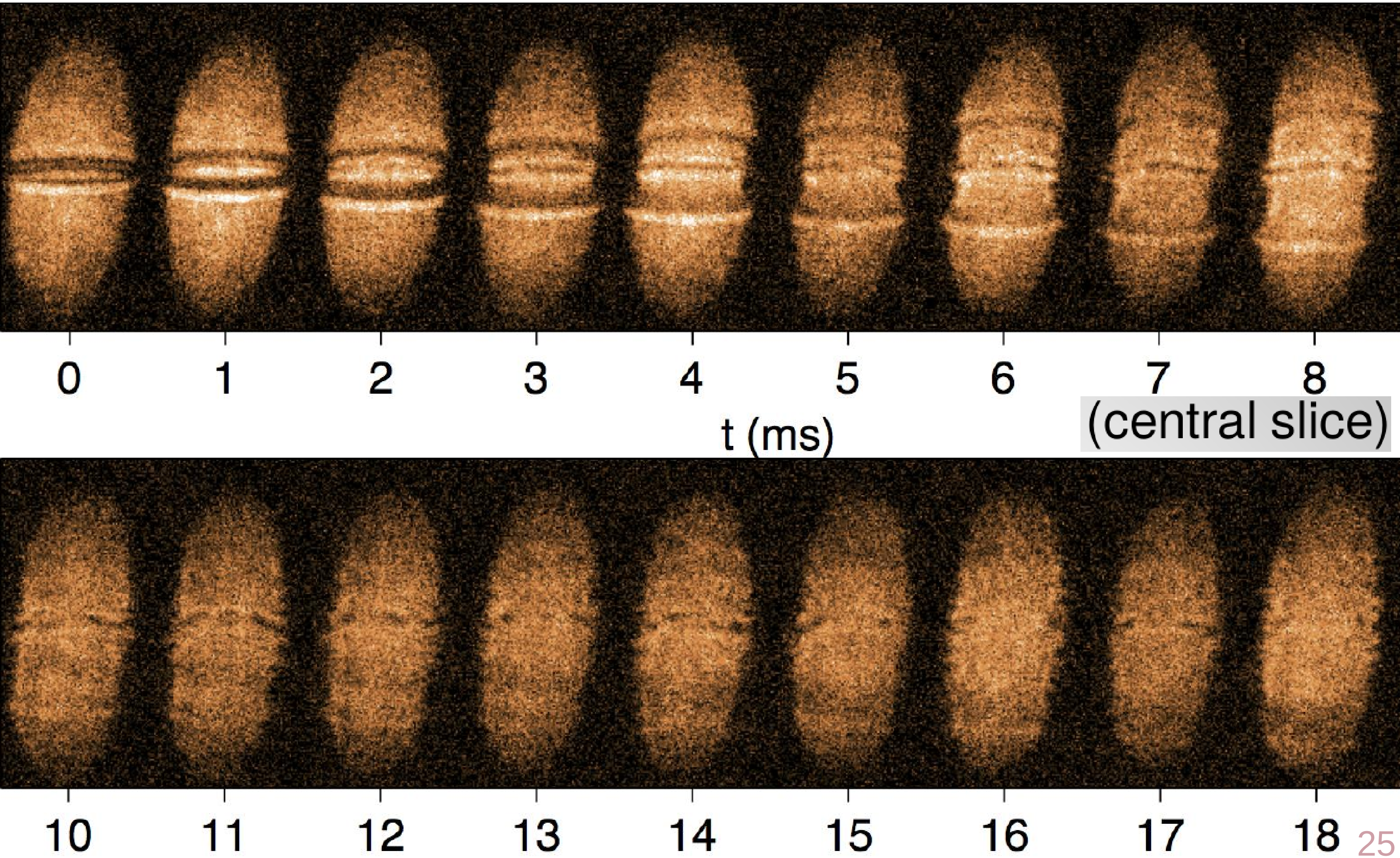
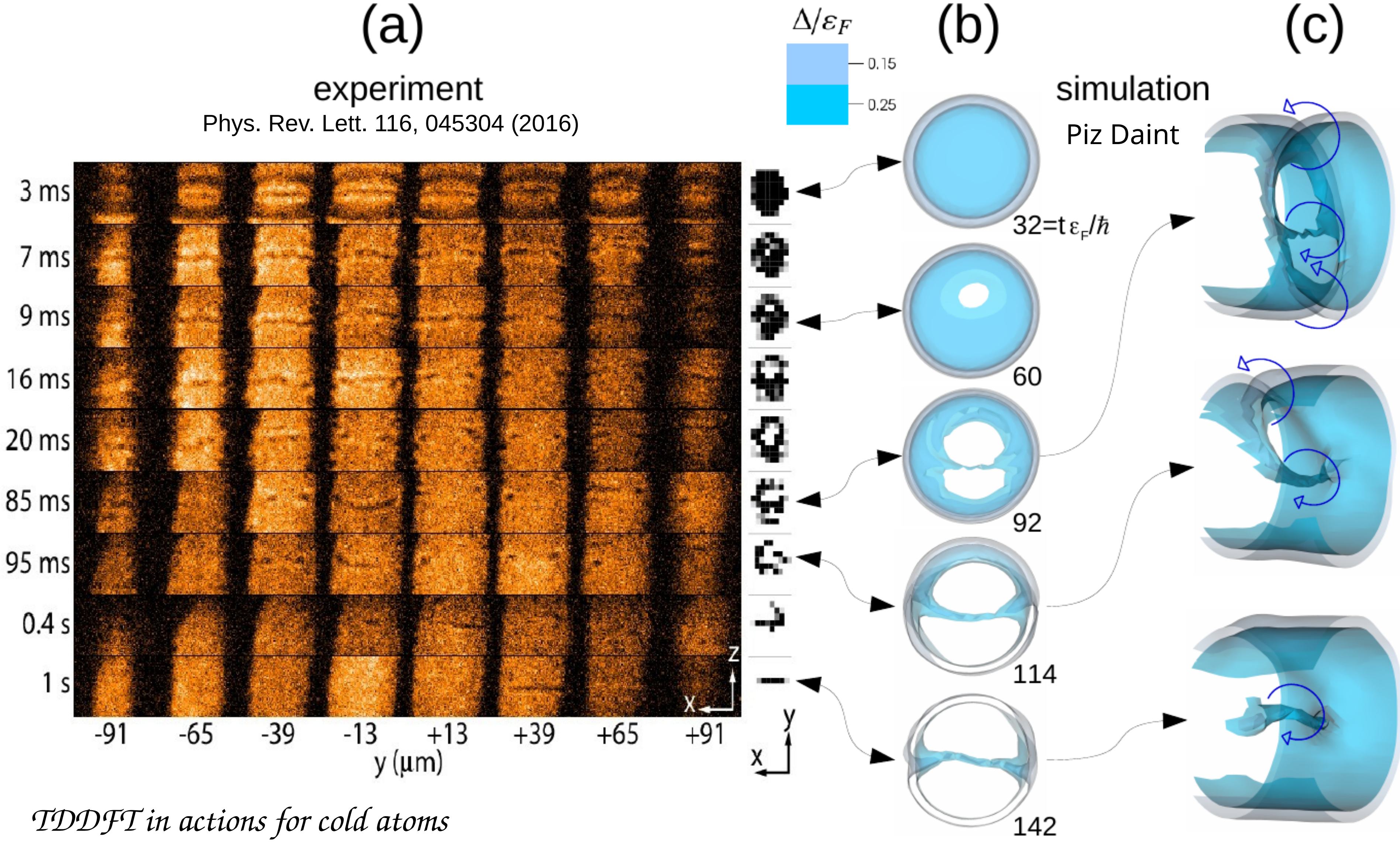


Fig. from
PRL 113, 065301(2014)

Experimental results – Cascade of Solitary Waves

Figures taken from: M. Zwierlein talk, (http://en.sif.it/activities/fermi_school/mmxiv)
School of Physics E. Fermi – Quantum Matter at Ultralow Temperatures Varenna, July 9th , 2014



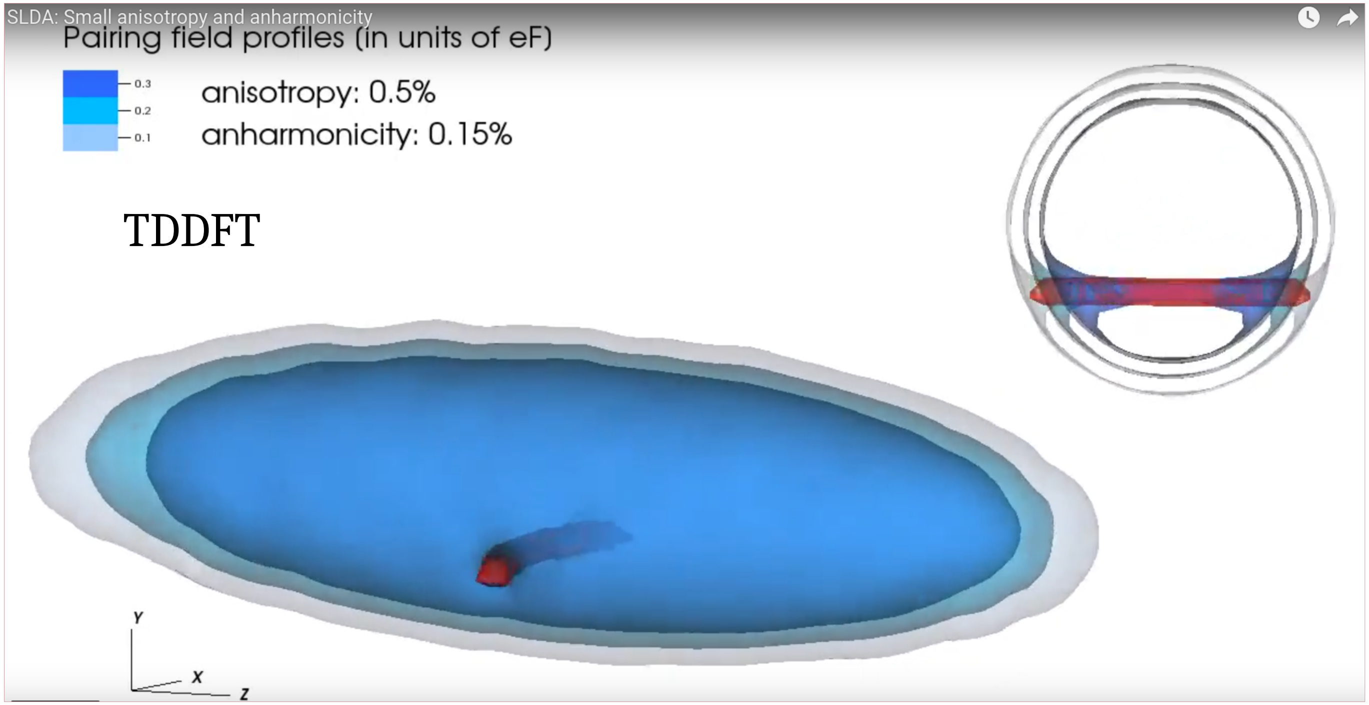


TDDFT in actions for cold atoms
→ remarkable agreement between theory and data!

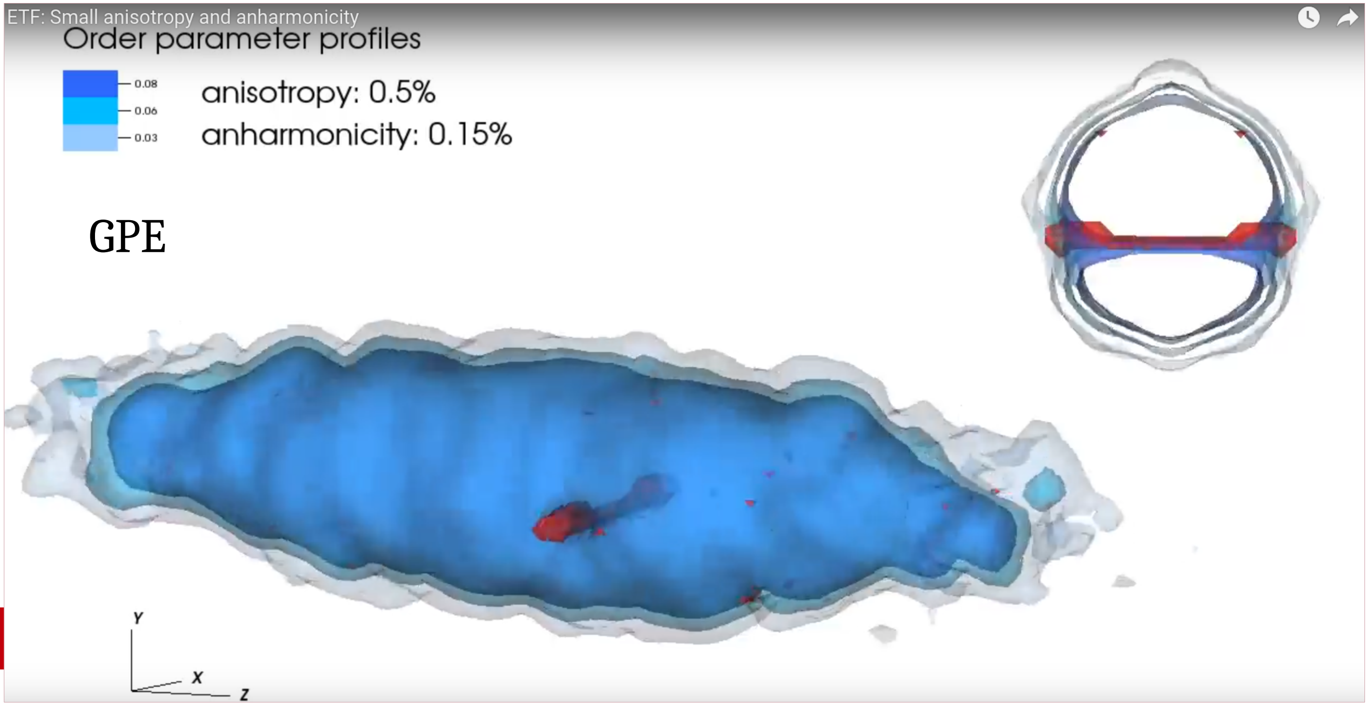
No adjusting parameters to the experiment!

Movie 2

For other comparison of TDDF vs exp see:
https://wslda.fizyka.pw.edu.pl/index.php/Gallery

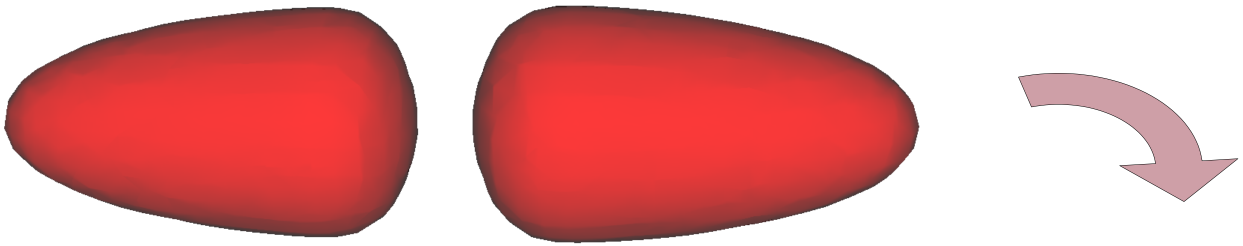


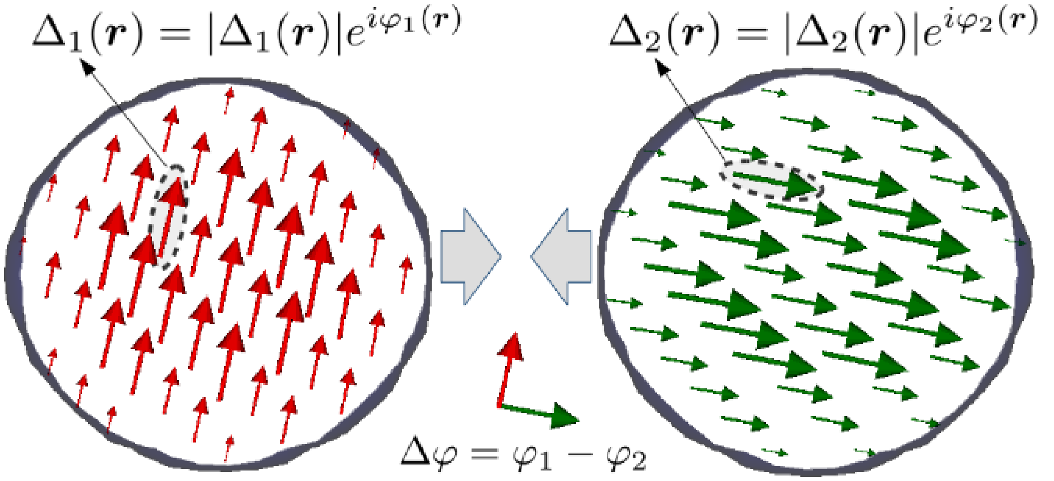
Note: *we find that effective approaches, like GPE, are able to predict correctly the final state, however the states in-between are different from TDDFT predictions.*



From:
G. Wlazłowski, A. Bulgac,
M.M. Forbes, K.J. Roche,
Phys. Rev. A 91, 031602(R) (2015).

“Pairing” as the dynamical field in nuclear physics?





Remarks:

- The phase φ (gauge angle) has well defined meaning only for systems with broken U(1) symmetry: N↔φ conjugate variables!
- In nuclear experiments the phase cannot be controlled.
- Possible signal should be extracted after averaging over the phase differences.
- Proper way of “averaging” is by projecting on a good particle number.

$$P^N = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{i\varphi(\hat{N}-N)}$$

Estimate of energy scales → energy needed to excite the "soliton"

➢ The additional energy cost (derived from Ginzburg-Landau theory)

$$E_j \sim \int_{V_j} \frac{\hbar^2}{2m_*} |\nabla \Psi|^2 \, d\boldsymbol{r}$$

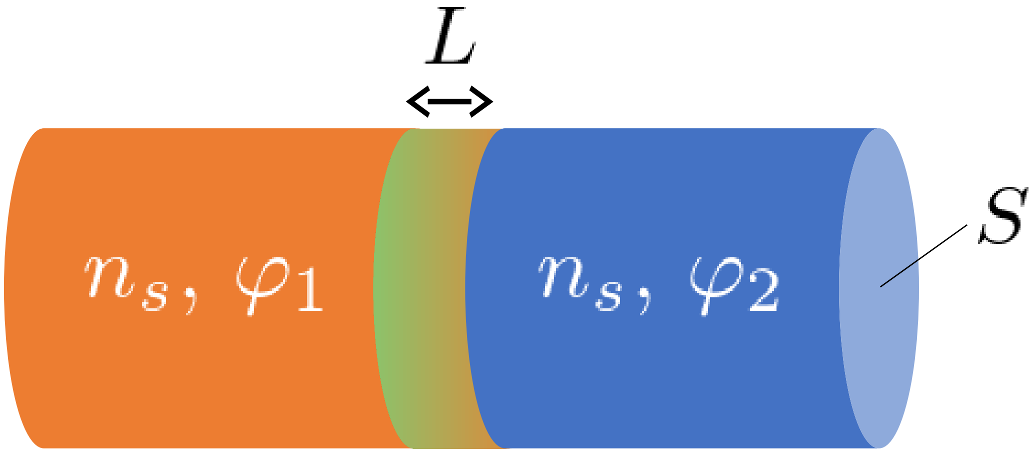
($\Psi \leftrightarrow$ order parameter)

$$\Psi = \sqrt{n_s} e^{i\varphi}$$

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\phi}{2}$$

The energy does not depend on the absolute value of Δ!

e.g.) $S = \pi R^2$,
$L \sim R = 6$ fm,
$n_s = 0.08$ fm^{-3}
→ $E \sim 30$ MeV



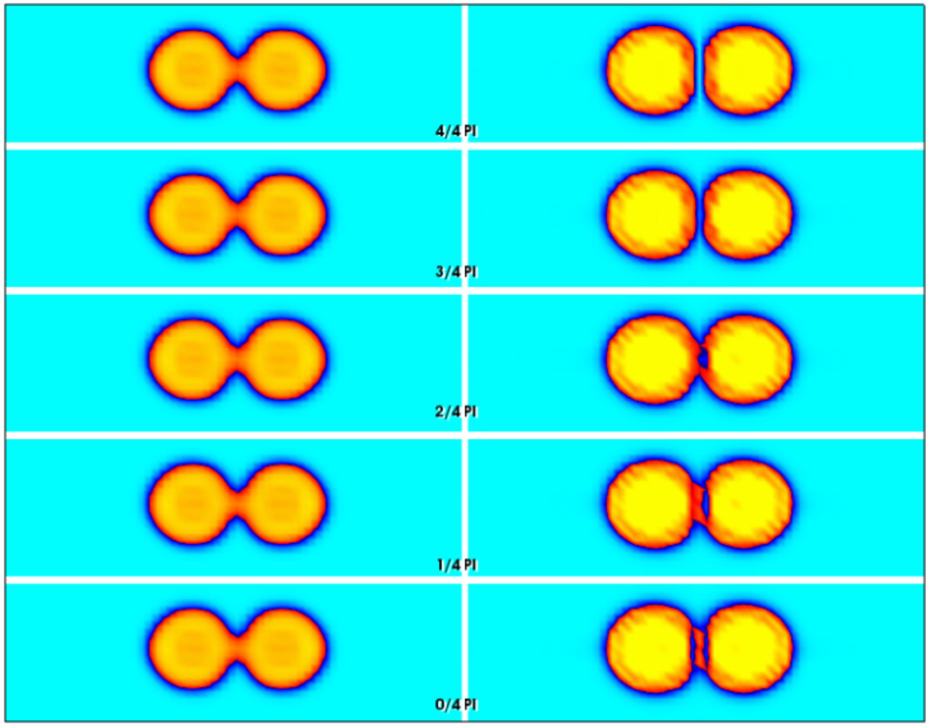
S: Attaching area

L: Length scale over which the phase varies

n_s: Superfluid density

"^{240}Pu+^{240}Pu" head-on collisions (E/V_{Bass}=1.1, E=980MeV)

Movie 3



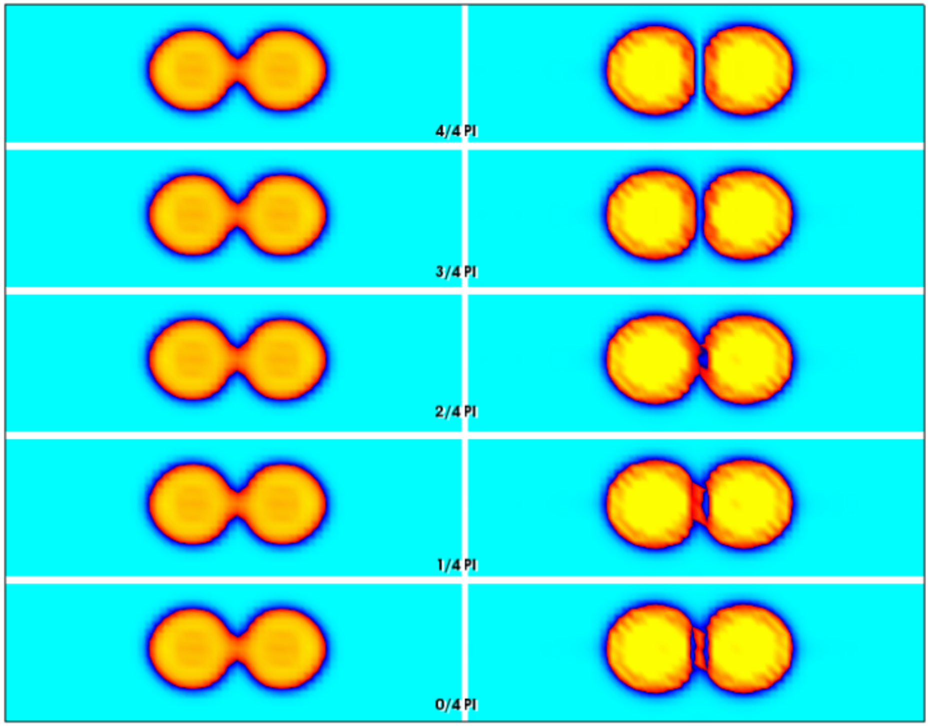
From: P. Magierski, K. Sekizawa, G. Wlazłowski,
Phys. Rev. Lett. 119, 042501 (2017)

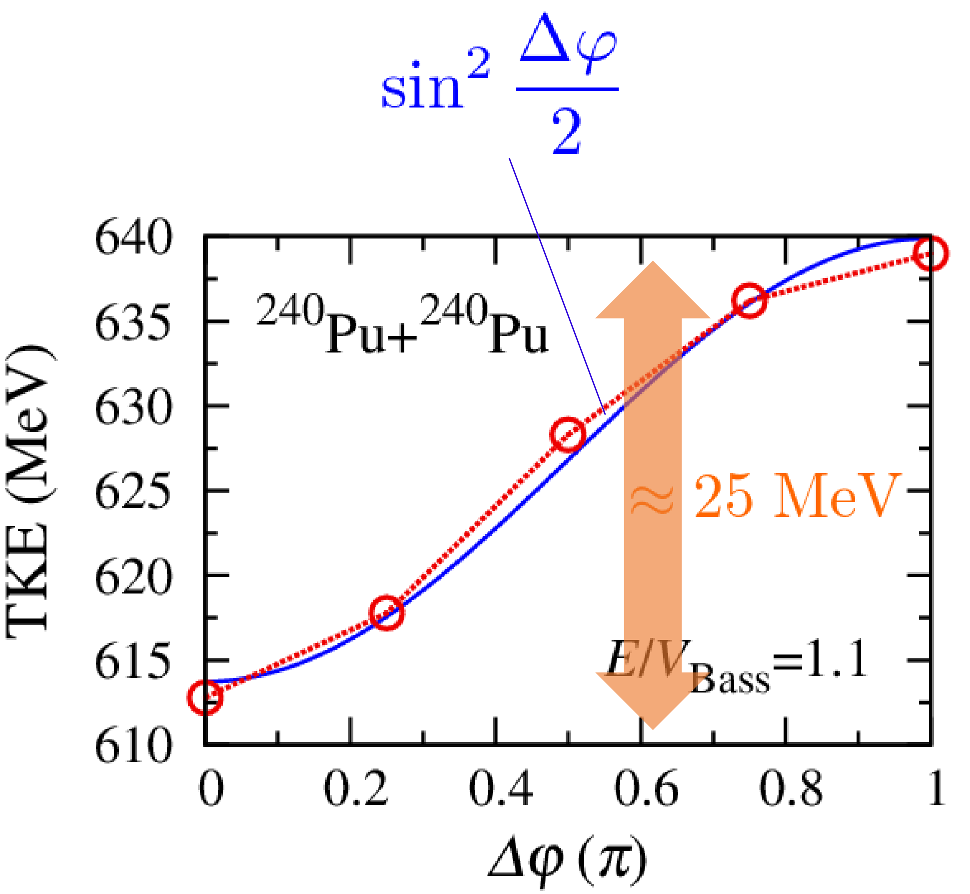
To simplify calcs:

- We used Fayans EDF (FaNDF^0)
  S.A. Fayans, JETP Letters 68, 169 (1998);
- We neglected spin-orbit term;
- These are "^{A}X nuclei" in sense that these objects have requested number of protons and neutrons (average value).

“^{240}Pu+^{240}Pu” head-on collisions (E/V_{Bass}=1.1, E=980MeV)

Movie 3





From: P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119, 042501 (2017)

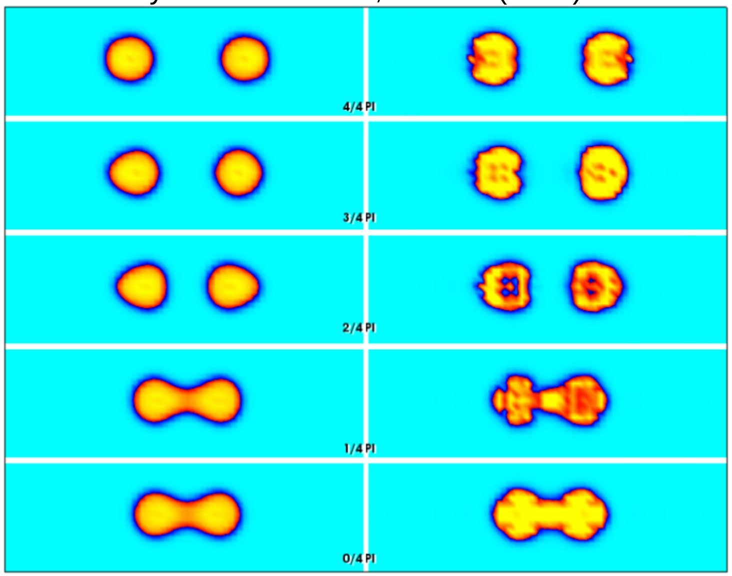
The phase difference changes **kinetic energy** of the fragments

Fusion reaction: $^{90}Zr+^{90}Zr$

Movie 4

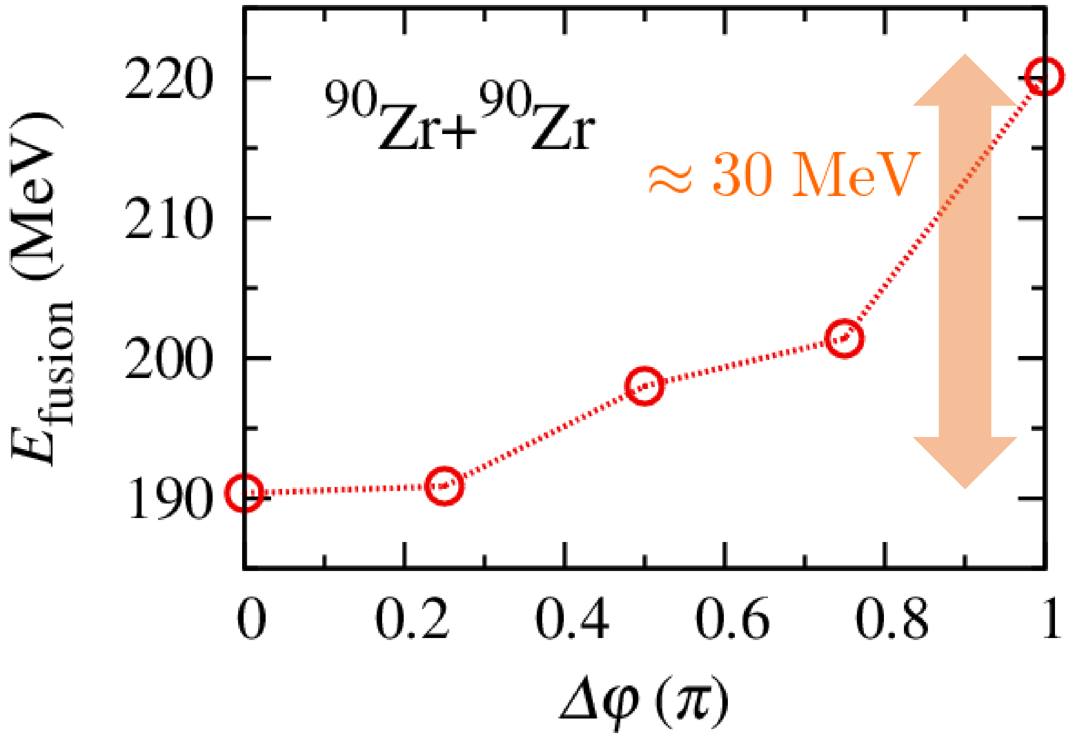
$E=1.0V_{Bass}$ (E=191MeV)

From: P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119, 042501 (2017)



Fusion reaction is **suppressed** by the phase difference

*E_{fusion}: the lowest energy at which fusion reaction is observed



Related studies:
Y. Hashimoto and G. Scamps, *Gauge angle dependence in TDHFB calculations of $^{20}O+^{20}O$ head-on collisions with the Gogny interaction*, Phys. Rev. C 94, 014610 (2016).

Examining empirical evidence of the effect of superfluidity on the fusion barrier

Guillaume Scamps
Phys. Rev. C **97**, 044611 – Published 18 April 2018

In order to describe the fusion barrier, three parameters are defined, the centroid barrier,

$$B_0 = \frac{m_1^B}{m_0^B}, \tag{5}$$

the fusion radius, defined in order to normalize the barrier distribution,

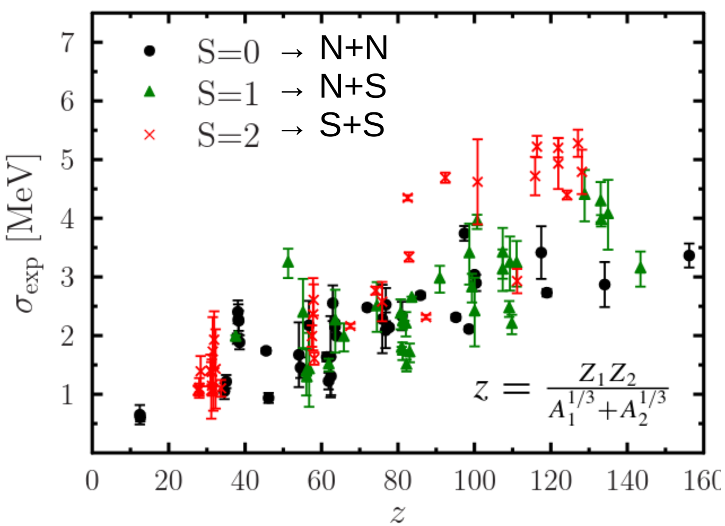
$$R_B = \sqrt{\frac{m_0^B}{\pi}}, \tag{6}$$

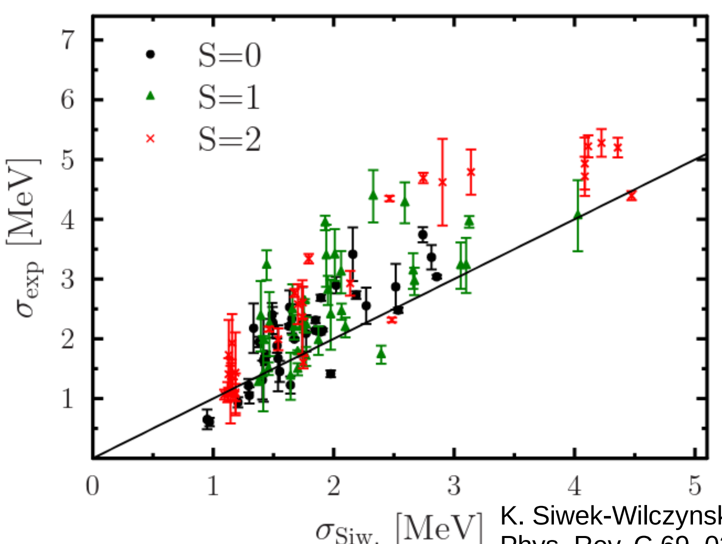
and the barrier width,

$$\sigma_B = \sqrt{\frac{m_2^B}{m_0^B} - \left(\frac{m_1^B}{m_0^B}\right)^2}. \tag{7}$$

These three parameters are computed from the moment of the barrier distribution,

$$m_n^B = \int_0^{E_M} B^n \frac{d^2}{dE^2}(E\sigma(E))\bigg|_{E=B} dB. \tag{8}$$



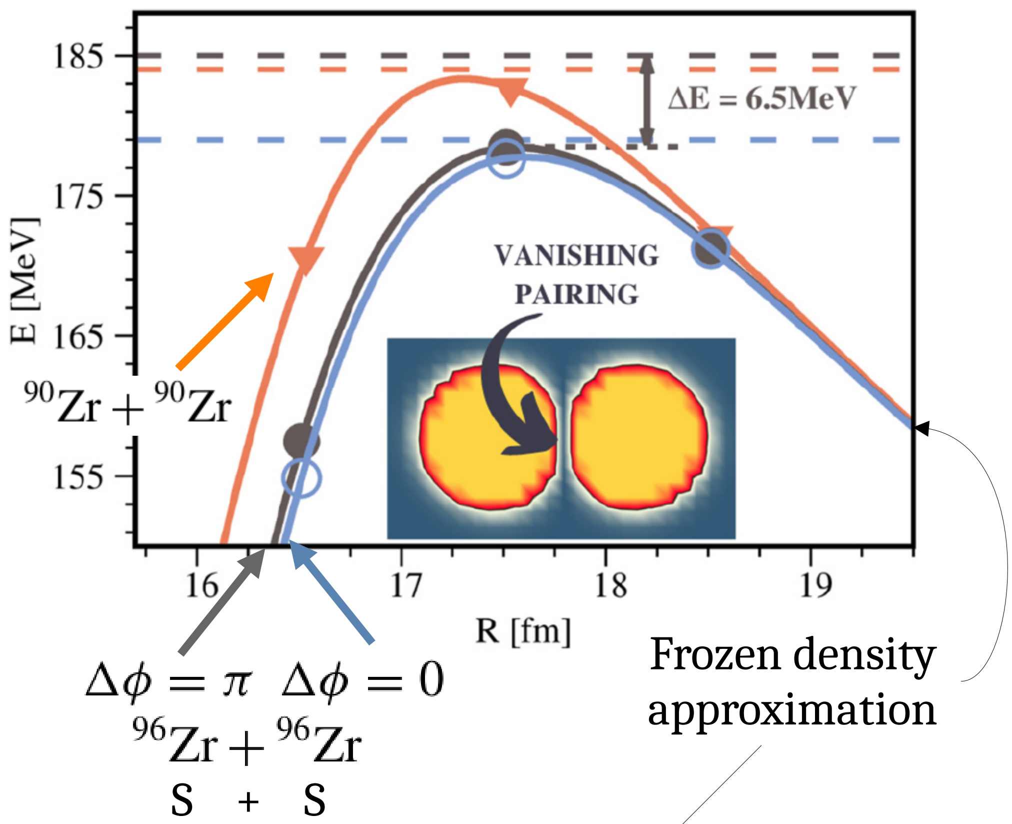


K. Siwek-Wilczynska and J. Wilczynski, Phys. Rev. C 69, 024611 (2004).

This method is applied to 115 fusion reactions and compared to a model that does not include the expected effect of superfluidity. An enhancement of the fluctuations of the barrier of about 1 MeV is found in several reactions between superfluid nuclei. This result proves that the effect predicted by TDHFB calculation is real. Nevertheless, this empirical result is in contradiction with the idea of a very strong effect of superfluidity in the fusion barrier.

Simulations with (complete) Skyrme SkM*

P. Magierski, A. Makowski, M.C. Barton, K. Sekizawa, and G. Wlazłowski
Phys. Rev. C 105, 064602 (2022)



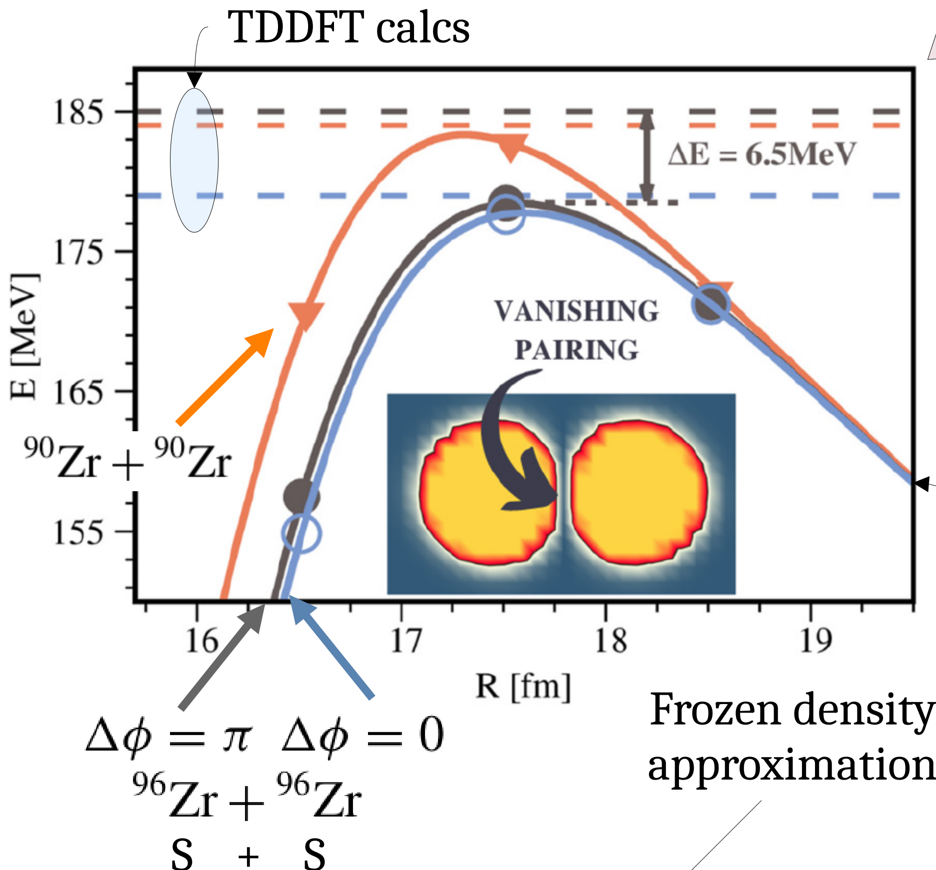
- In the frozen density approximation the dynamical effects are neglected ...
- ... density of each fragment is fixed to be its ground-state one...
- ... the contribution from the pairing fields were also taken into account:

$$\Delta_{q,\mathrm{tot}}(\mathbf{r}) = \Delta_{q,1}(\mathbf{r} - R/2) + \Delta_{q,2}(\mathbf{r} + R/2)$$

Simulations with (complete) Skyrme SkM*

P. Magierski, A. Makowski, M.C. Barton, K. Sekizawa, and G. Wlazłowski
Phys. Rev. C 105, 064602 (2022)

The effect is of dynamic origin, which cannot be grasped in the static calculations.



	$\overline{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	ΔE_s
^{90}Zr	$\overline{\Delta}_n = 0.00$	184	184	0
	$\overline{\Delta}_p = 0.09$			
^{96}Zr	$\overline{\Delta}_n = 1.98$	179	185	6
	$\overline{\Delta}_p = 0.32$			
	$\overline{\Delta}_n = 2.44$	178	187	9
	$\overline{\Delta}_p = 0.33$			
	$\overline{\Delta}_n = 2.94$	178	187	9
	$\overline{\Delta}_p = 0.34$			

Movie 5 Movie 6

Much weaker effect than predicted within simplified clacs!

- In the frozen density approximation the dynamical effects are neglected ...
- ... density of each fragment is fixed to be its ground-state one...
- ... the contribution from the pairing fields were also taken into account:

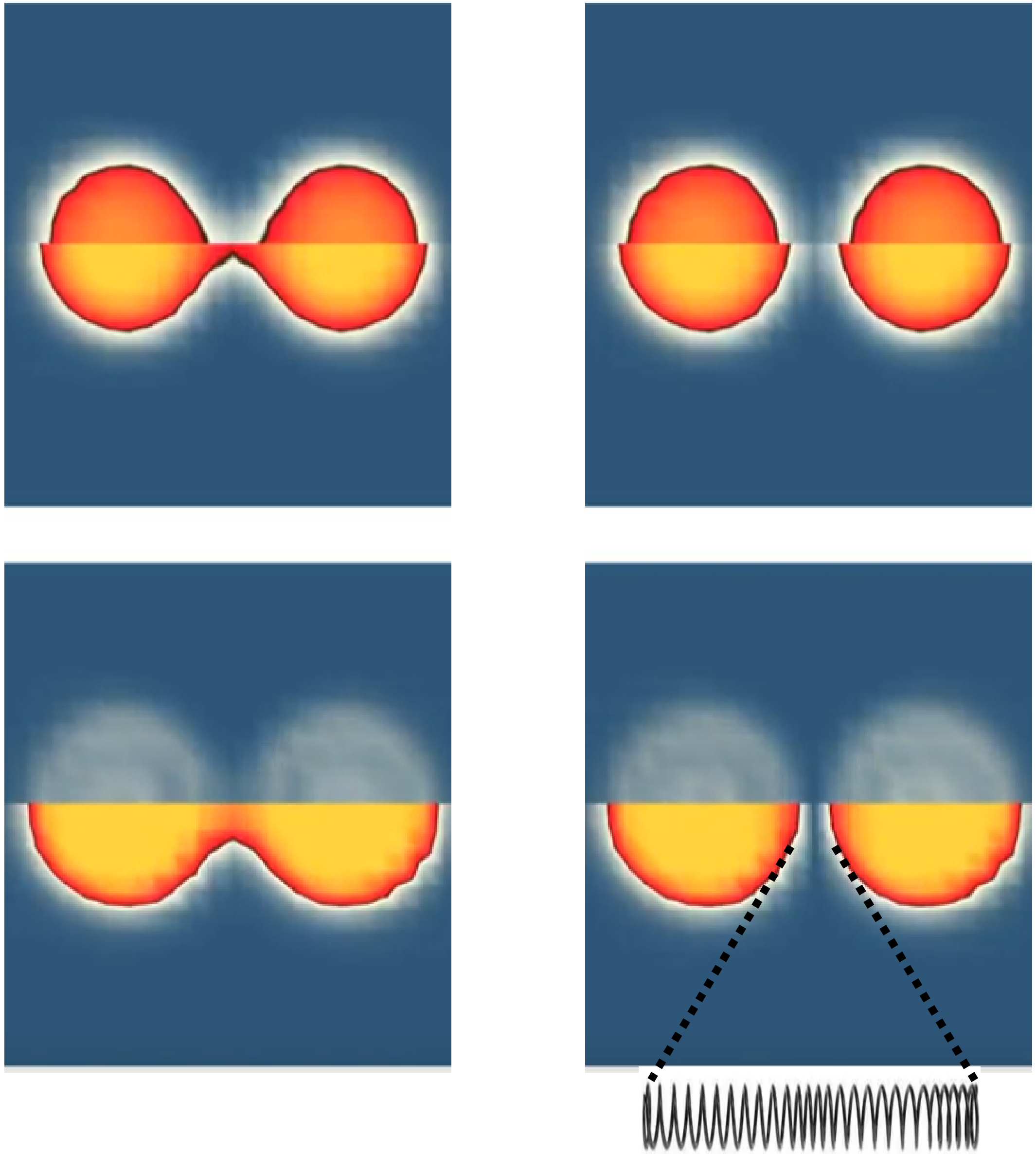
$$\Delta_{q,\text{tot}}(\mathbf{r}) = \Delta_{q,1}(\mathbf{r} - R/2) + \Delta_{q,2}(\mathbf{r} + R/2)$$

$^{96}Zr + ^{96}Zr$,
E_{cm}=178MeV

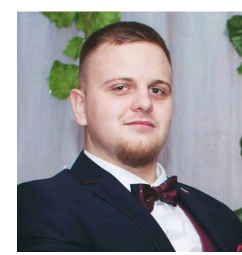
Δφ=0 Δφ=π

density:
p
n

paring field:
p
n



Related talk:
Wiktor Kragiel



P. Magierski, A. Makowski, M.C. Barton, K. Sekizawa, and G. Wlazłowski
Phys. Rev. C 105, 064602 (2022)

Pairing emergence during the collision

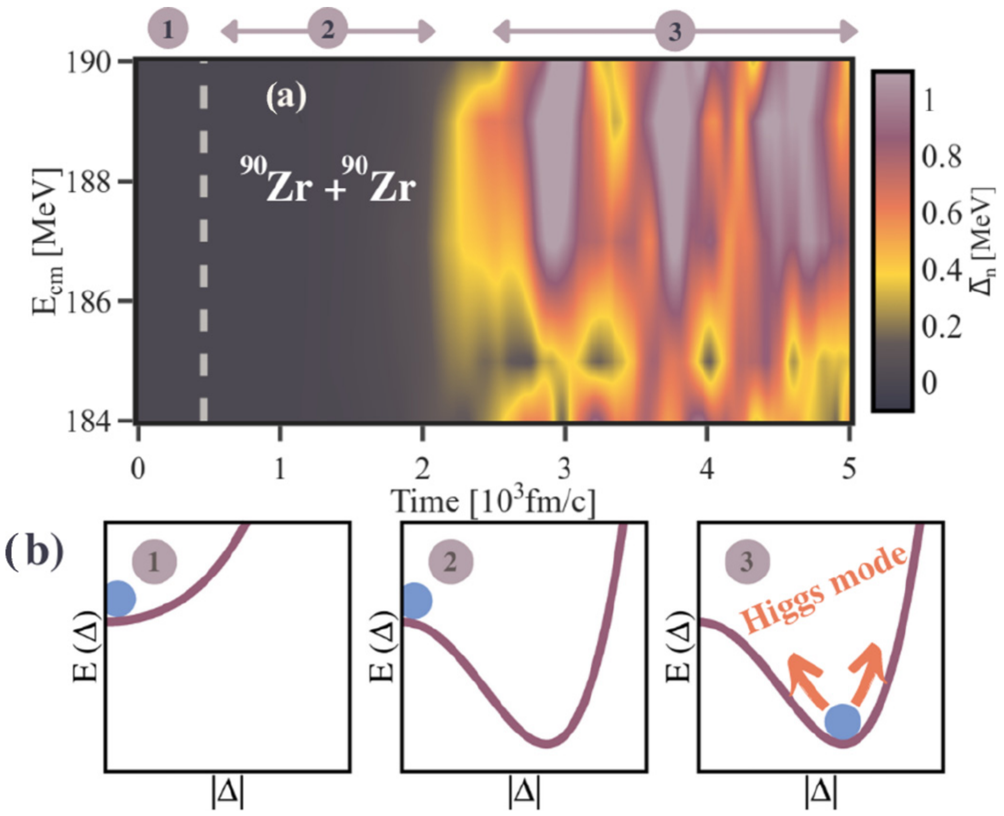
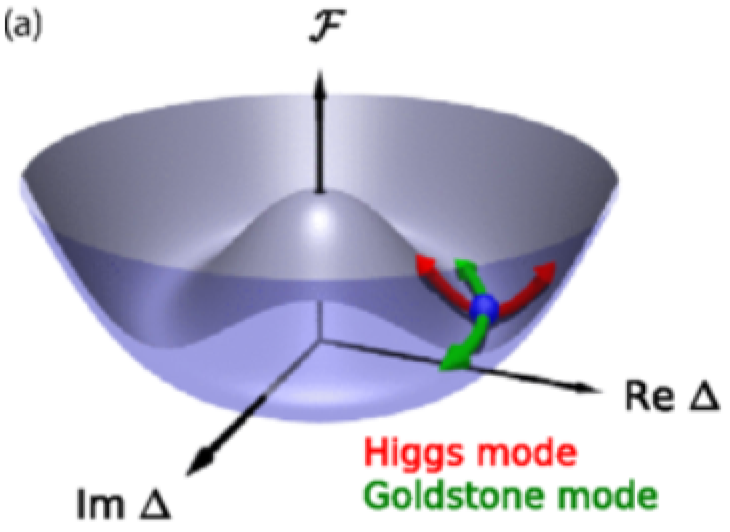


Fig. From: A. F. Kemper, et.al., Phys. Rev. B 92, 224517 (2015).



Related talk:
Andrzej Makowski

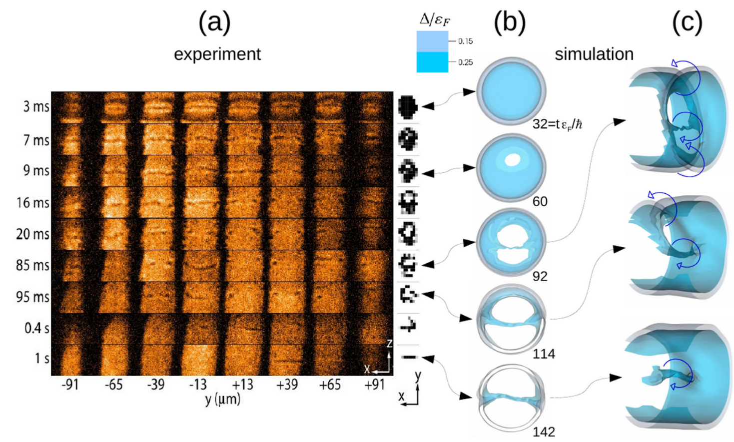
Large-amplitude oscillations of pairing field
→ similarity to the pairing Higgs mechanism.

SUMMARY

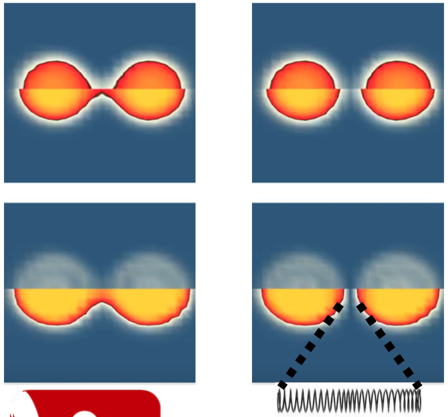
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}U_{\uparrow n}(\mathbf{r},t)\\ U_{\downarrow n}(\mathbf{r},t)\\ V_{\uparrow n}(\mathbf{r},t)\\ V_{\downarrow n}(\mathbf{r},t)\end{pmatrix}=\begin{pmatrix}h_{\uparrow\uparrow}(\mathbf{r},t) & h_{\uparrow\downarrow}(\mathbf{r},t) & 0 & \Delta_{\uparrow\downarrow}(\mathbf{r},t)\\ h_{\downarrow\uparrow}(\mathbf{r},t) & h_{\downarrow\downarrow}(\mathbf{r},t) & \Delta_{\downarrow\uparrow}(\mathbf{r},t) & 0\\ 0 & \Delta^*_{\downarrow\uparrow}(\mathbf{r},t) & -h^*_{\uparrow\uparrow}(\mathbf{r},t) & -h^*_{\uparrow\downarrow}(\mathbf{r},t)\\ \Delta^*_{\uparrow\downarrow}(\mathbf{r},t) & 0 & -h^*_{\downarrow\uparrow}(\mathbf{r},t) & -h^*_{\downarrow\downarrow}(\mathbf{r},t)\end{pmatrix}\begin{pmatrix}U_{\uparrow n}(\mathbf{r},t)\\ U_{\downarrow n}(\mathbf{r},t)\\ V_{\uparrow n}(\mathbf{r},t)\\ V_{\downarrow n}(\mathbf{r},t)\end{pmatrix}$$

- Ultracold atomic gases can be used as a playground for testing predictive power of TDDFT...

  … still, complex nature of nuclear interaction/energy density functional rises many questions about trustability of TDDFT in context of nuclear reactions.



- Solitonic excitations:
  - *their dynamics have been properly reproduced by TDDFT for ultracold Fermi gases...*
  - … and analog of them is predicted to be present for nuclear reactions.



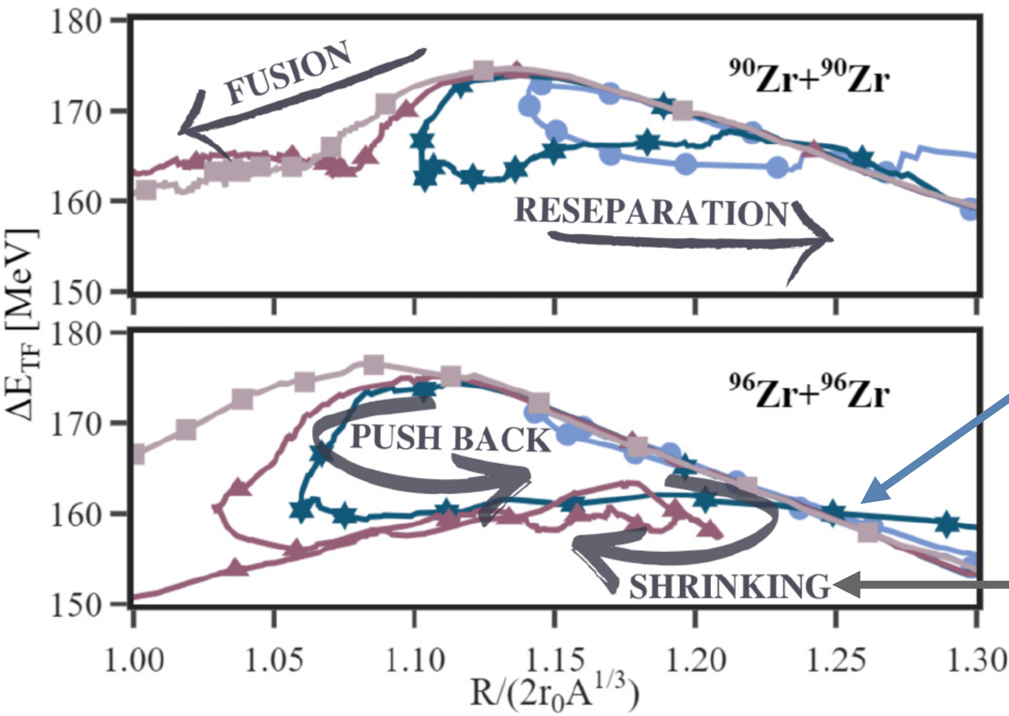
- Nuclear analog of “solitonic excitation” impacts:
  - *Barrier for fussion;*
  - *TKE of fragments;*
  - *Dynamics of neck formation;*

Collaborators: P. Magierski, A. Makowski, W. Kragiel (WUT); K. Sekizawa (Tokyo); A. Bulgac (UW);

Contact:
gabriel.wlazlowski@pw.edu.pl
http://wlazlowski.fizyka.pw.edu.pl

Thank you!

Appendix



Movie 7

Total excitation energies (TXEs) in $^{96}Zr + ^{96}Zr$ at c.m. energies just below the threshold for capture.

	$\overline{\Delta}_q$ (MeV)	$E_{\text{c.m.}}$ (MeV)	TXE (MeV) $\Delta\phi = 0$	TXE (MeV) $\Delta\phi = \pi$
^{96}Zr	$\overline{\Delta}_n = 1.98$, $\overline{\Delta}_p = 0.32$	178	37	25
	$\overline{\Delta}_n = 2.44$, $\overline{\Delta}_p = 0.33$	177	34	10
	$\overline{\Delta}_n = 2.94$, $\overline{\Delta}_p = 0.34$	177	34	8

$$E_{\text{TF}} = E_{\text{Coul}} + \int d^3\mathbf{r}\left[\sum_{q=n,p}\frac{\hbar^2}{2m_q}\tau_q^{\text{TF}} + \sum_{t=0,1}\left(C_t^{\rho}\rho_t^2 + C_t^{\Delta\rho}\rho_t\nabla^2\rho_t + C_t^{\tau}\rho_t\tau_t^{\text{TF}} + C_t^{\gamma}\rho_t^2\rho_0^{\gamma}\right)\right]$$

$$\tau_q^{\text{TF}} = \frac{3}{5}(3\pi^2)^{2/3}\rho_q^{5/3} + \frac{1}{36}\frac{(\nabla\rho_q)^2}{\rho_q} + \frac{1}{3}\nabla^2\rho_q$$

It indicates that the presence of pairing alters the process of neck formation.

Phase diagram of ultracold Fermi gas

Experiments:

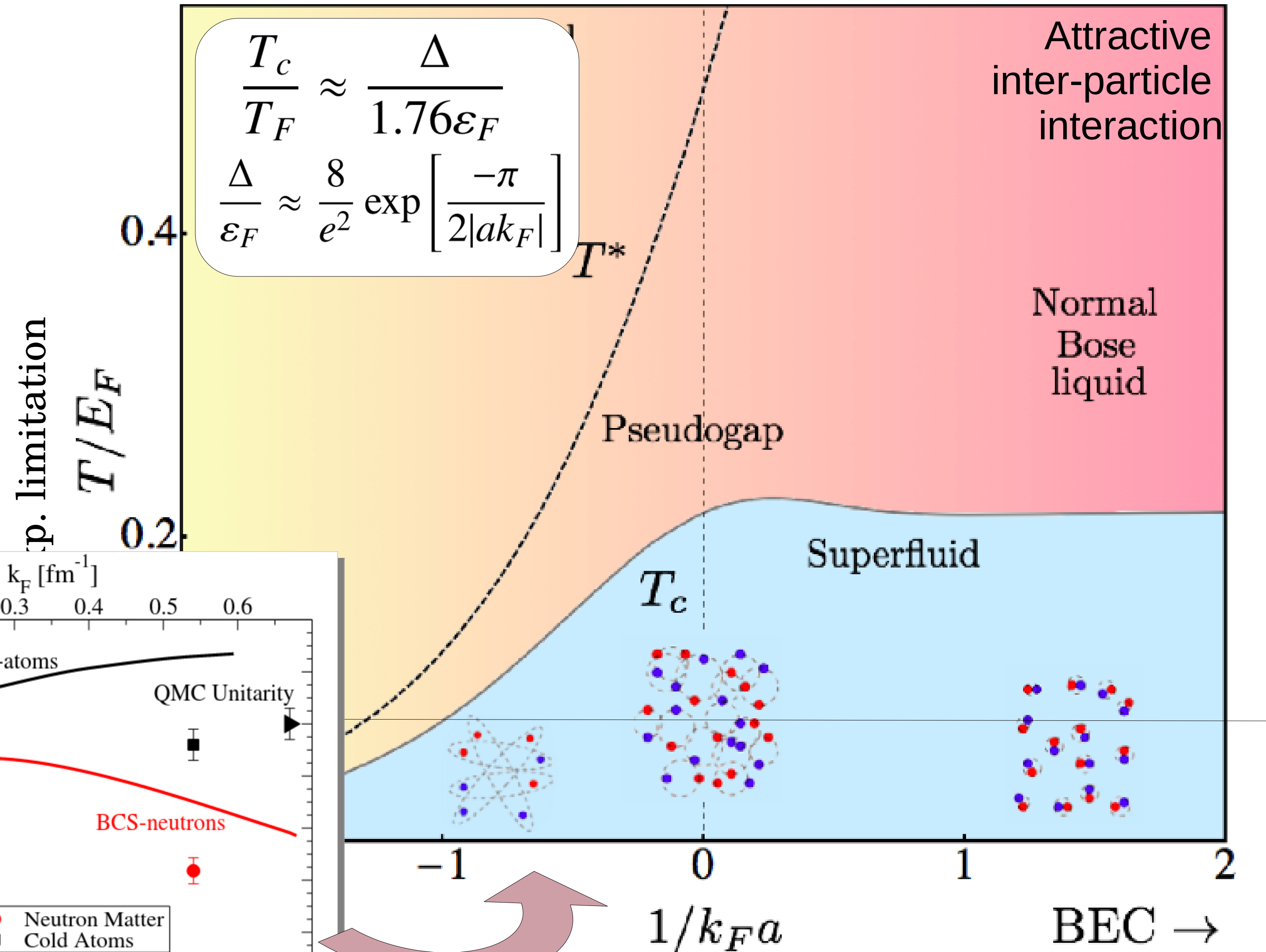
$$\frac{T}{T_F} \gtrsim 0.05$$

↓

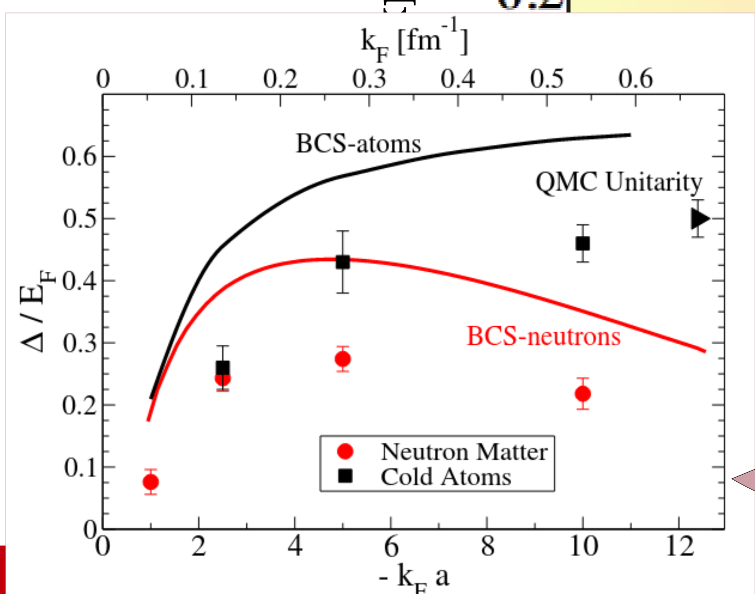
$$|ak_F| \gtrsim 1$$

$$\frac{T_c}{T_F} \approx \frac{\Delta}{1.76\varepsilon_F}$$

$$\frac{\Delta}{\varepsilon_F} \approx \frac{8}{e^2} \exp\left[\frac{-\pi}{2|ak_F|}\right]$$



A. Gezerlis and J. Carlson
Phys. Rev. C 77, 032801(R) (2008)



Josephson effect

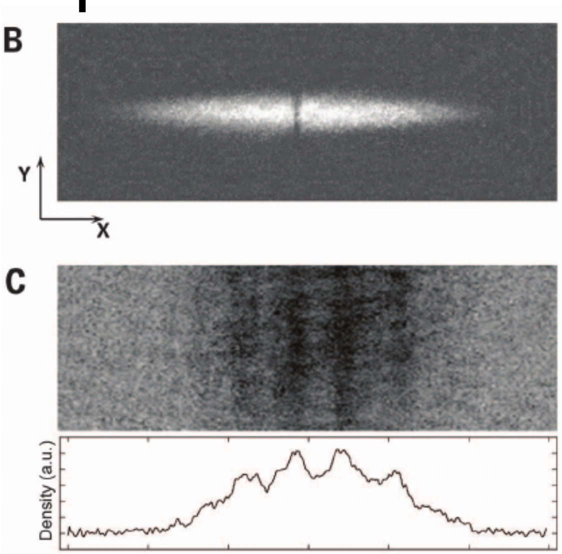
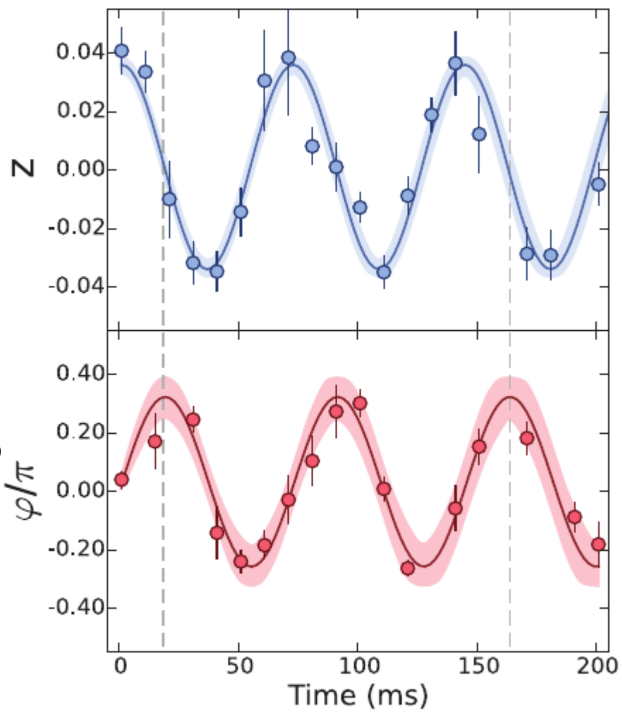
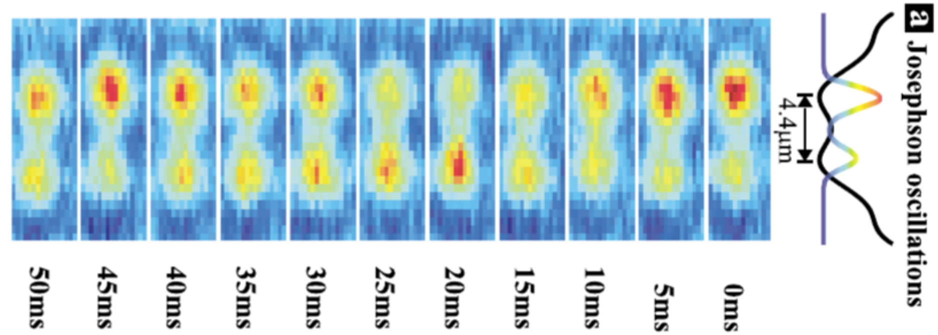


Fig. 1. Josephson junction between two ultracold fermionic superfluids. (**A**) Sketch of the experimental



Figs from: G. Valtolina, et. al., Science 350, 1505 (2015)



Figs from: M. Albiez, et. al., Phys. Rev. Lett. 95, 010402 (2005)

$$i\hbar\frac{\partial\psi_1}{\partial t} = \mathcal{E}_1\psi_1 - \mathcal{T}\psi_2,$$

$$i\hbar\frac{\partial\psi_2}{\partial t} = \mathcal{E}_2\psi_2 - \mathcal{T}\psi_1,$$

$$\psi_j = \sqrt{n_j}\,\mathrm{e}^{i\varphi_j}$$

Flow of particles maximized when phase diff. is $\pi/2$

$$\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t} = \frac{2\mathcal{T}}{\hbar}\sqrt{n_1 n_2}\,\sin(\varphi_2 - \varphi_1),$$

$$\frac{\partial(\varphi_2 - \varphi_1)}{\partial t} = \frac{\mathcal{E}_2 - \mathcal{E}_1}{\hbar} + \frac{\mathcal{T}}{\hbar}\frac{n_1 - n_2}{\sqrt{n_1 n_2}}\cos(\varphi_2 - \varphi_1).$$

SLDA-type functional

Dimensionless functional parameters

$$\{A_\lambda, B_\lambda, C_\lambda\}$$

Densities $n(\boldsymbol{r}), \tau(\boldsymbol{r}), \nu(\boldsymbol{r})$ are defined via $[u_\eta(\boldsymbol{r},t), v_\eta(\boldsymbol{r},t)]^T$

$$E = \int \mathcal{E}[n(\boldsymbol{r}), \tau(\boldsymbol{r}), \nu(\boldsymbol{r})]d\boldsymbol{r}$$

$$\mathcal{E} = A_\lambda \frac{\tau}{2} + \frac{3}{5}B_\lambda n \varepsilon_{\mathrm{F}} + \frac{C_\lambda}{n^{1/3}}|\nu|^2$$

Kinetic term — Potential term — Pairing term

MINIMIZATION

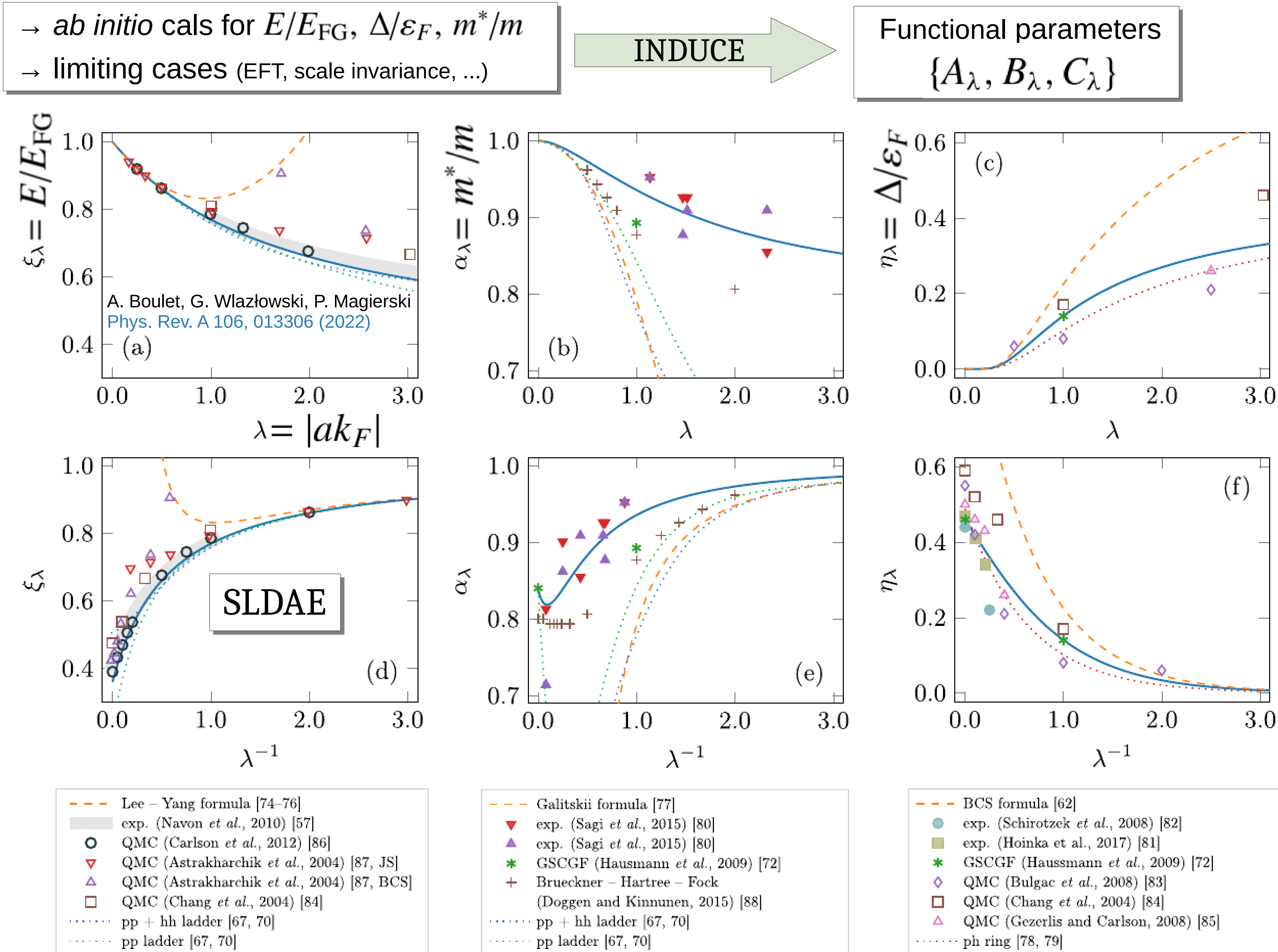
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_\eta(\mathbf{r},t) \\ v_\eta(\mathbf{r},t)\end{pmatrix} = \mathcal{H}_{\mathrm{SLDA}}\begin{pmatrix} u_\eta(\mathbf{r},t) \\ v_\eta(\mathbf{r},t)\end{pmatrix}$$

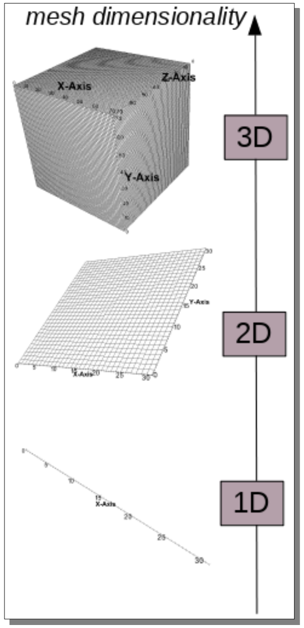
A. Bulgac, M.M. Forbes
Phys. Rev. A 75, 031605(R) (2007)

A. Boulet, G. Wlazłowski, P. Magierski
Phys. Rev. A 106, 013306 (2022)

BdG	ASLDA Asymmetric SLDA, a→∞	SLDAE SLDA Extended, p=0
$A_\lambda \to 1$	$A_\lambda \to A[p(\boldsymbol{r})]$	$A_\lambda \to A[ak_F(\boldsymbol{r})]$
$B_\lambda \to 0$	$B_\lambda \to B[p(\boldsymbol{r})]$	$B_\lambda \to B[ak_F(\boldsymbol{r})]$
$C_\lambda \to -\frac{4\pi\hbar^2}{(3\pi^2)^{1/3}m}ak_F$	$C_\lambda \to C[p(\boldsymbol{r})]$	$C_\lambda \to C[ak_F(\boldsymbol{r})]$

$$p(\boldsymbol{r}) = \frac{n_\uparrow(\boldsymbol{r}) - n_\downarrow(\boldsymbol{r})}{n_\uparrow(\boldsymbol{r}) + n_\downarrow(\boldsymbol{r})}$$


→ ab initio cals for E/E_{FG}, Δ/ε_F, m^*/m
→ limiting cases (EFT, scale invariance, ...)
INDUCE
Functional parameters
$\{A_\lambda, B_\lambda, C_\lambda\}$
$\xi_\lambda = E/E_{FG}$
$\alpha_\lambda = m^*/m$
$\eta_\lambda = \Delta/\varepsilon_F$
$\lambda = |ak_F|$
λ
ξ_λ
α_λ
η_λ
λ^{-1}
(a)
(b)
(c)
(d)
(e)
(f)
A. Boulet, G. Wlazłowski, P. Magierski
Phys. Rev. A 106, 013306 (2022)
SLDAE
Lee – Yang formula [74–76]
exp. (Navon et al., 2010) [57]
QMC (Carlson et al., 2012) [86]
QMC (Astrakharchik et al., 2004) [87, JS]
QMC (Astrakharchik et al., 2004) [87, BCS]
QMC (Chang et al., 2004) [84]
pp + hh ladder [67, 70]
pp ladder [67, 70]
Galitskii formula [77]
exp. (Sagi et al., 2015) [80]
exp. (Sagi et al., 2015) [80]
GSCGF (Hausmann et al., 2009) [72]
Brueckner – Hartree – Fock
(Doggen and Kinnunen, 2015) [88]
pp + hh ladder [67, 70]
pp ladder [67, 70]
BCS formula [62]
exp. (Schirotzek et al., 2008) [82]
exp. (Hoinka et al., 2017) [81]
GSCGF (Haussmann et al., 2009) [72]
QMC (Bulgac et al., 2008) [83]
QMC (Chang et al., 2004) [84]
QMC (Gezerlis and Carlson, 2008) [85]
ph ring [78, 79]



→ BCS-BEC crossover
→ spin-imbalanced systems
→ mass-imbalanced systems
→ finite temperature formalism

Ongoing extensions:
→ Bose-Fermi mixtures
→ Fermi-Fermi mixtures (like nuclear systems: protons+neutrons)

Warsaw University of Technology | W-SLDA Toolkit

http://wslda.fizyka.pw.edu.pl/

W-SLDA Toolkit

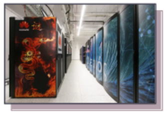
Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

$$i\hbar\frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$


High Performance Computing

can run on “small” computing clusters as well as leadership supercomputers
(depending on the problem size)

NVIDIA CUDA AMD ROCm open source

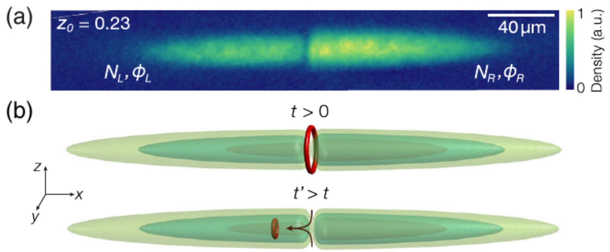
PHYSICS.WUT

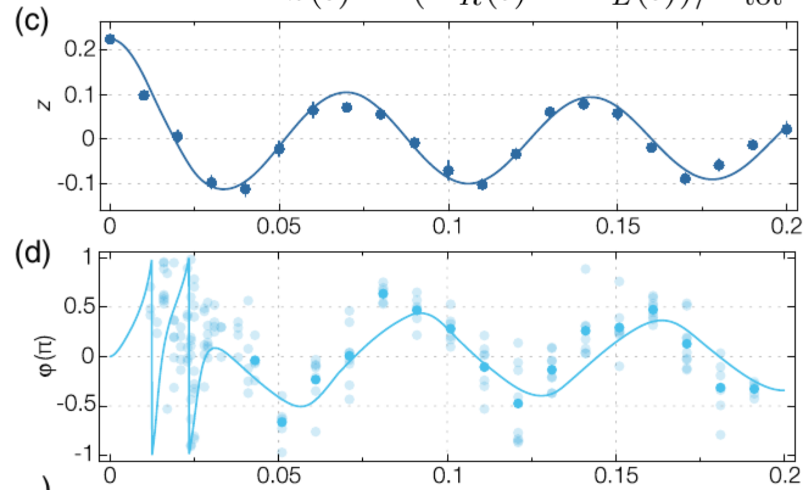
Application #1: Fermionic Josephson Junction

Inspired by LENS ^{6}Li setup (G. Roati's group):

[1] G. Valtolina, et.al., Science **350**, 1505, (2015);
[2] A. Burchianti, et.al., Phys. Rev. Lett. **120**, 025302 (2018)
[3] K. Xhani, et.al., Phys. Rev. Lett. **124**, 045301 (2020)

$z(t) = (N_R(t) - N_L(t))/N_{\rm tot}$





Figs from [2]

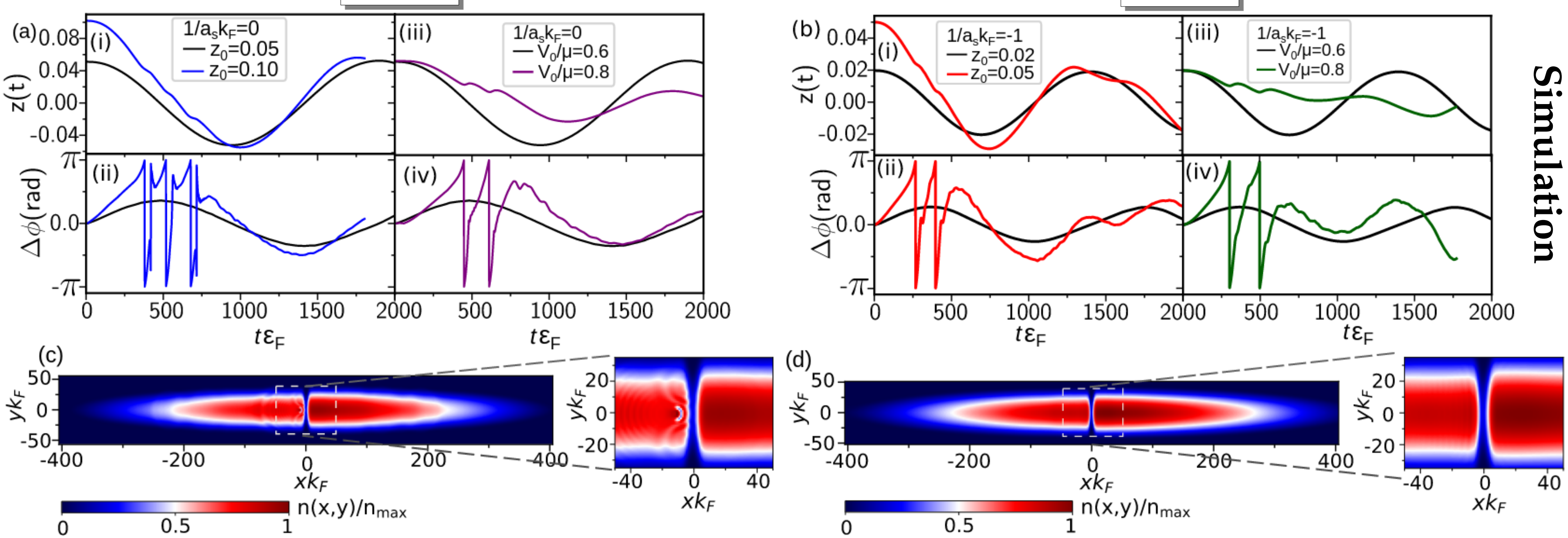
$\Delta\phi(t) \equiv \phi_L(t) - \phi_R(t)$ t [ms]

Experiment

G. Wlazłowski, et.al., arXiv:2207.06059

UFG

BCS



Simulation