Quantum turbulence in superfluid Fermi gases: results of numerical modeling

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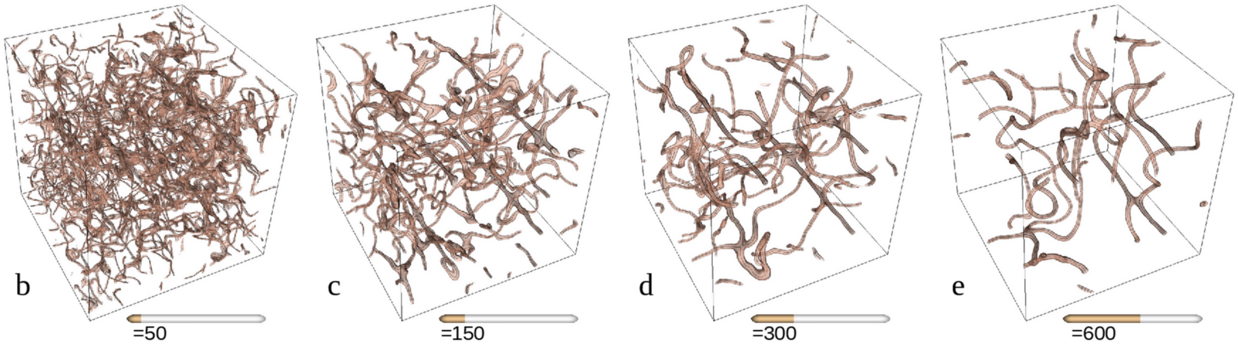
University of Washington



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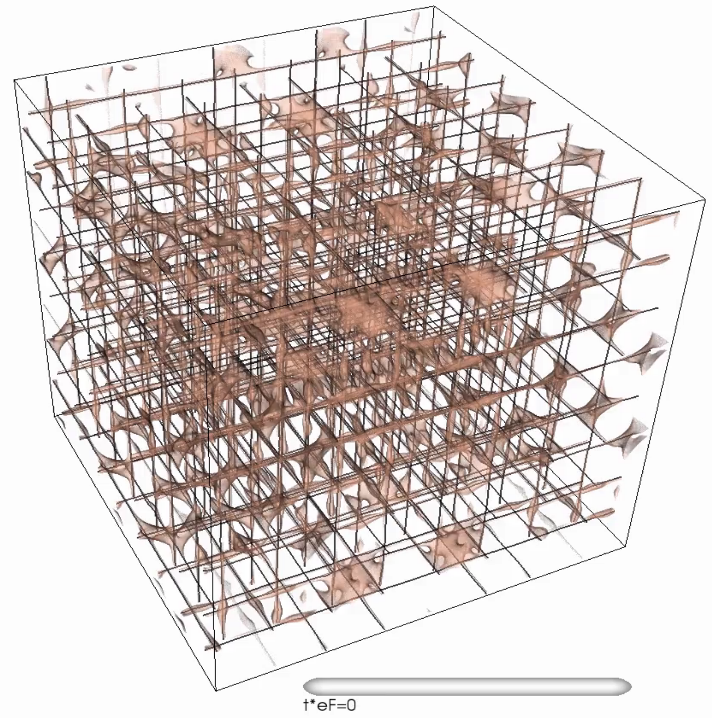
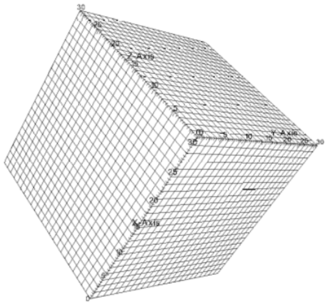
Nonequilibrium phenomena in strongly-correlated ultracold matter
Erice-Sicily, 9 – 15 May 2024

Quantum turbulence



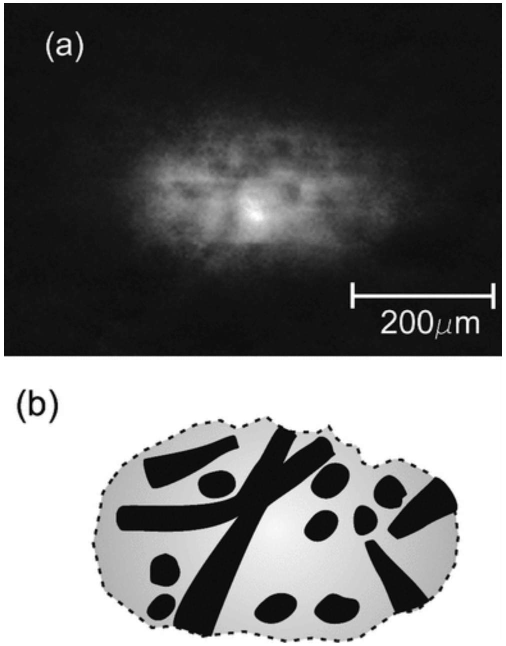
System: *unitary Fermi gas (spin-symmetric)*
number of atoms = 26,790
Method: *Time-Dependent Density Functional Theory*
PNAS Nexus, pgae160 (2024)

Computation
on spatial grid



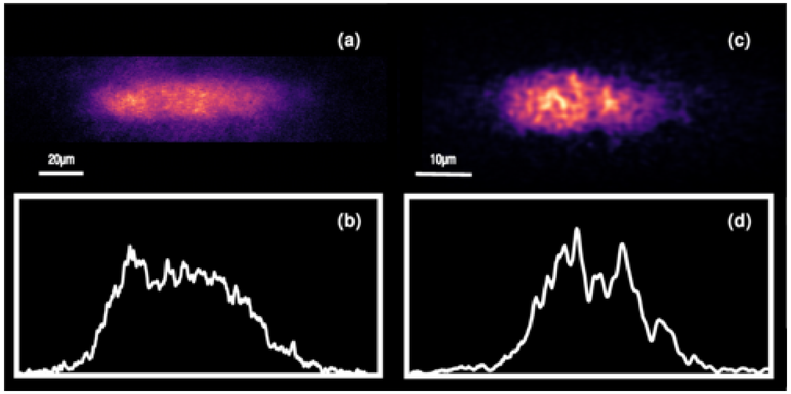
(the largest system in 3D we considered had 108,532 atoms)

Quantum turbulence in Bose systems

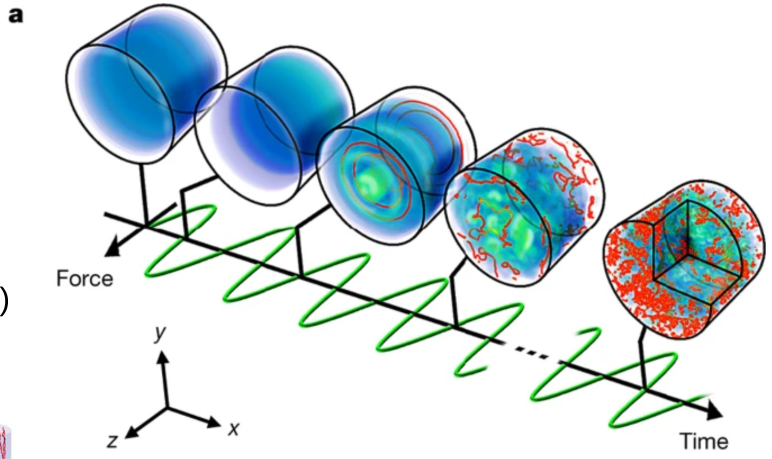


E. A. L. Henn, J. A. Seman, G. Roati, K. M. F. Magalhães,
and V. S. Bagnato, Phys. Rev. Lett. 103, 045301 (2009)

…



H. A. J. Middleton-Spencer, A. D. G. Orozco, L. Galantucci,
M. Moreno, N. G. Parker, L. A. Machado, V. S. Bagnato,
and C. F. Barenghi, Phys. Rev. Research 5, 043081 (2023)



Nir Navon, Alexander L. Gaunt, Robert P. Smith & Zoran Hadzibabic
Nature 539, p. 72–75 (2016)

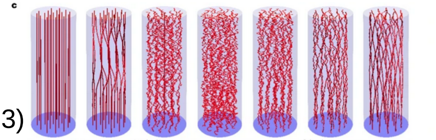
Reviews:

- L. Madeira, et al., Ann. Rev. of Cond. Mat. Phys., 11 (2020)
- M.C. Tsatsos, et al., Phys. Rep. 622, 1 (2016).
- M. Tsubota, et al., J. Low. Temp. Phys. 188, 119 (2017)

…

… *superfluid helium* …

… J. T. Mäkinen, et.al.,
Nat. Phys. 19, 898 (2023)



An active field of research!

Superfluidity across BEC-BCS crossover

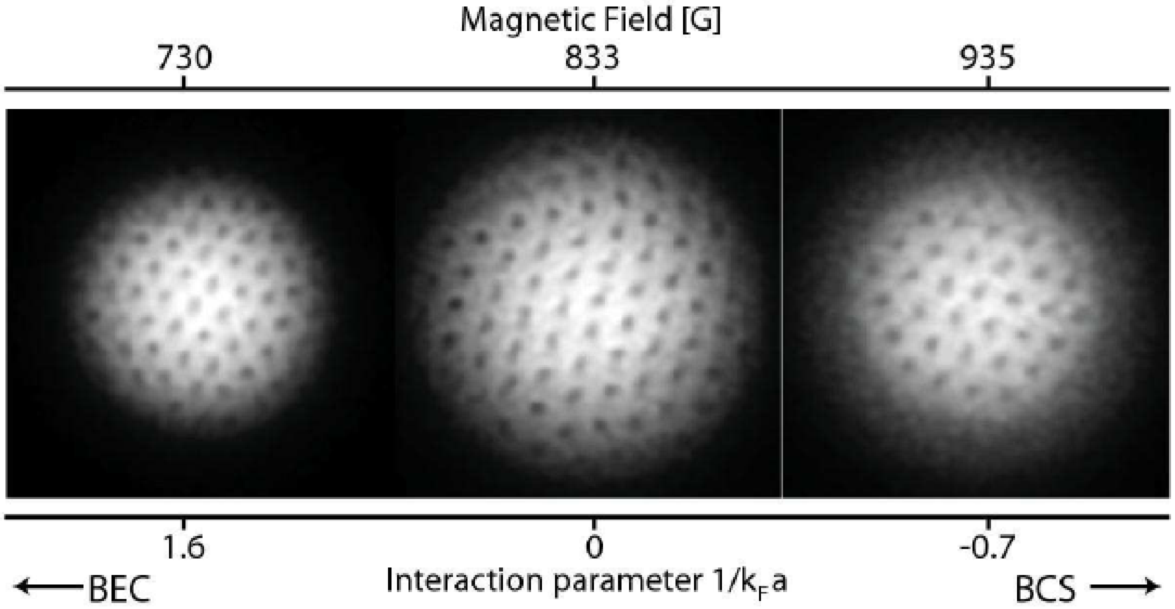


FIG. 36 Vortex lattice in a rotating gas of ^{6}Li precisely at the Feshbach resonance and on the BEC and BCS side. Reprinted with permission from Zwierlein *et al.* (2005).

M. W. Zwierlein, J. R. Abo-Shaeer, A. Schirotzek, C. H. Schunck, and W. Ketterle, Nature 435, 1047 (2005).

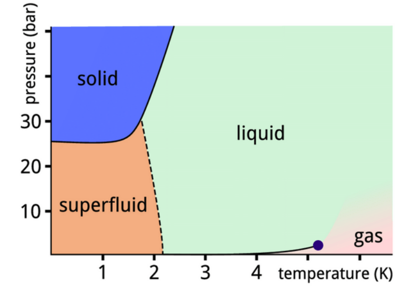
Comparing Bose & Fermi superfluids

L=0, S=0
(s-wave superfluidity)

→ the simplest form of superfluidity in Fermi systems

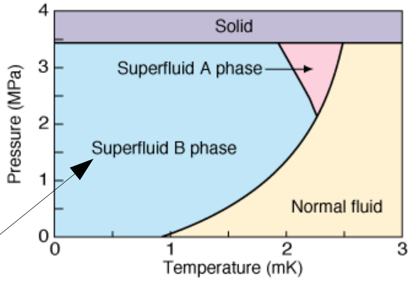
→ BEC-BCS crossover allows to make **direct comparisons within the same system!**

^{4}He



↕

^{3}He



L=1, S=1
(p-wave, spin-triplet)

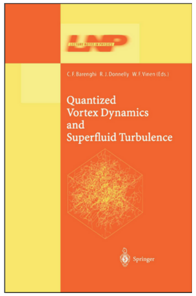
→ additional level of complication

Scientific question:
What is **impact of quantum statistics** on superfluid (turbulent) dynamics?

Macroscopic
Hall-Vinen-Bekharevich-Khalatnikov (HVBK) equations

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\frac{1}{\rho}\nabla P - \frac{\rho_s}{\rho_n} S \nabla T + \nu_n \nabla^2 \mathbf{v}_n + \frac{\rho_s}{\rho}\mathbf{F},$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s = -\frac{1}{\rho}\nabla P + S \nabla T + \mathbf{T} - \frac{\rho_n}{\rho}\mathbf{F},$$


Quantized
Vortex Dynamics
and
Superfluid Turbulence

C. F. Barenghi R. J. Donnelly
W. F. Vinen (Eds.),
*QuantizedVortex Dynamics
and Superfluid Turbulence*,
Springer

Mesoscopic
Vortex Filament Model

$$\mathbf{f}_D + \mathbf{f}_M = 0,$$

$$\mathbf{f}_M = \rho_s \mathit{\Gamma} \mathbf{s}' \times (\mathbf{v}_L - \mathbf{v}_{s,tot}),$$

$$\mathbf{f}_D = -\alpha \rho_s \mathit{\Gamma} \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n - \mathbf{v}_{s,tot})] - \alpha' \rho_s \mathit{\Gamma} \mathbf{s}' \times (\mathbf{v}_n - \mathbf{v}_{s,tot}),$$

Microscopic
Gross–Pitaevskii equation (GPE)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi - m\mathcal{E}\psi + V_0 \psi |\psi|^2,$$

Macroscopic
Hall-Vinen-Bekharevich-Khalatnikov (HVBK) equations

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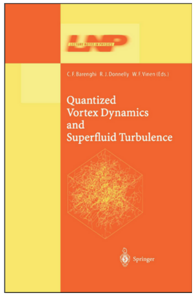

C. F. Barenghi R. J. Donnelly W. F. Vinen (Eds.), *QuantizedVortex Dynamics and Superfluid Turbulence*, Springer

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Microscopic (Bose)
Gross–Pitaevskii equation (GPE)

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - m\mathcal{E}\psi + V_0\psi|\psi|^2,$$

Microscopic (Fermi)
Bogolubov-de Gennes equations (BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_\eta(\mathbf{r},t) \\ v_\eta(\mathbf{r},t) \end{pmatrix} = \mathcal{H}_{\mathrm{BdG}}\begin{pmatrix} u_\eta(\mathbf{r},t) \\ v_\eta(\mathbf{r},t) \end{pmatrix}$$

$$\mathcal{H}_{\mathrm{BdG}} = \begin{pmatrix} h_\uparrow(\mathbf{r},t) - \mu_\uparrow & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h_\downarrow^*(\mathbf{r},t) + \mu_\downarrow \end{pmatrix}$$

The system is described as a collection of quasiparticles (a mixture of hole u_η and particle v_η).
In GPE formalism, only one state is considered, while in BdG, one needs to consider many of them!

→ High-Performance Computing (HPC)
← *we use this approach*
→ Construction of effective approaches like Local Phase Density Approximation [Strinati et. al.]

Synergy: theory & experiment

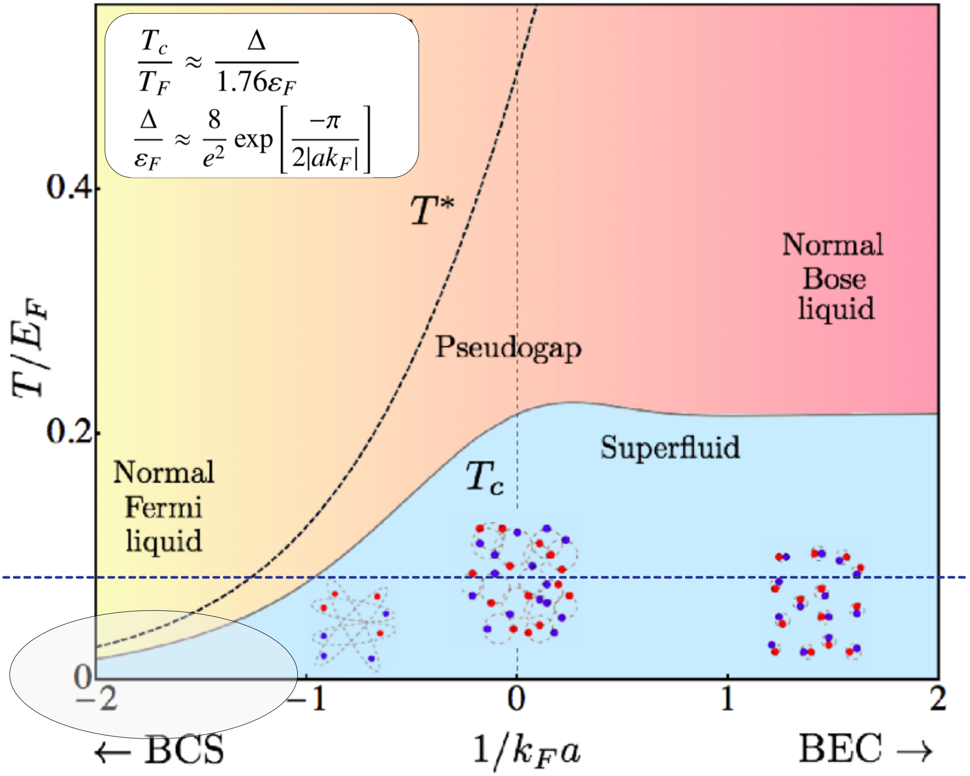
Experiments:

$$\frac{T}{T_F} \gtrsim 0.05$$

$$\downarrow$$

$$|ak_F| \gtrsim 1$$

~exp. limitation



Regime
of validity
of BdG theory →

(note: BdG for uniform
system= BCS theory)

Mean-field
Theory

Synergy: theory & experiment

Experiments:

$$\frac{T}{T_F} \gtrsim 0.05$$

$$\downarrow$$

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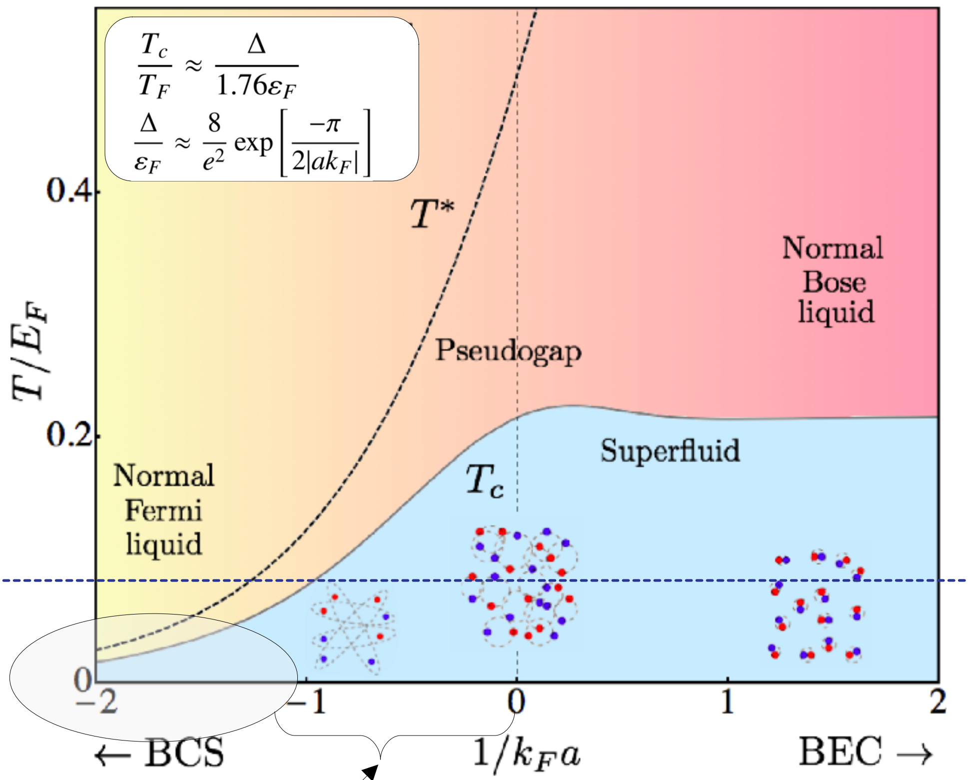

Mean-field Theory

Density Functional Theory

Regime of validity of BdG theory →

(note: BdG for uniform system= BCS theory)

DFT is, in principle, exact theory (due to Hohenberg-Kohn theorem)…

… in practice not – we need to construct the energy functional (no mathematical recipe how to derive it)

Nature 514, 550 (2014)

… Twelve papers on the top-100 list relate to it [DFT], *including 2 of the top 10.*

Many extensions: time-dependent formalism, finite temperature, normal/superconducting systems, non-relativistic/relativistic, …

General purpose framework

SLDA-type functional

Superfluid **L**ocal
Density **A**pproximation

$$\boxed{E_0 = \int \mathcal{E}[n_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}}$$

The Fermi-Dirac
distribution function

normal density

$$n_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

Denisties are **parametrized** via
Bogoliubov quasiparticle wave functions

kinetic density

$$\tau_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

quasiparticle = mixture of
hole particle

$$\varphi_\eta(\boldsymbol{r},t) = [u_\eta(\boldsymbol{r},t), v_\eta(\boldsymbol{r},t)]^T$$

current density

$$\boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} \mathrm{Im}[v_{n,\sigma}(\boldsymbol{r})\nabla v^*_{n,\sigma}(\boldsymbol{r})] f_\beta(-E_n),$$

$$\int \varphi^\dagger_\eta(\boldsymbol{r},t)\varphi_{\eta'}(\boldsymbol{r},t)\, d^3\boldsymbol{r} = \delta_{\eta,\eta'}$$

+ orthonormality condition (Pauli principle)

anomalous density

$$\nu(\boldsymbol{r}) = \frac{1}{2}\sum_{|E_n|<E_c} \left[u_{n,a}(\boldsymbol{r})v^*_{n,b}(\boldsymbol{r}) - u_{n,b}(\boldsymbol{r})v^*_{n,a}(\boldsymbol{r})\right] f_\beta(-E_n).$$

Additional density required by DFT theorem for systems with broken U(1) symmetry

Energy cut-off scale (need for regularization)

SLDA (and BdG) allows for solutions: **n≠0 and ν=0**

→ ***Cooper pair breaking*** → ***effectively normal component***

SLDA-type functional

$$\boxed{E_0 = \int \mathcal{E}[n_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}}$$

$$\Big\downarrow \text{minimization}$$

By construction minimization of the SLDA-type functional leads to equations that are mathematically equivalent to BdG or HFB equations

$$\boxed{\begin{pmatrix} h_\uparrow(\boldsymbol{r}) - \mu_\uparrow & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_\downarrow^*(\boldsymbol{r}) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix}}$$

$$h_\sigma = -\boldsymbol{\nabla} \frac{\delta E_0}{\delta \tau_\sigma} \boldsymbol{\nabla} + \frac{\delta E_0}{\delta n_\sigma} - \frac{i}{2} \left\{ \frac{\delta E_0}{\delta \boldsymbol{j}_\sigma}, \boldsymbol{\nabla} \right\}, \quad \Delta = -\frac{\delta E_0}{\delta \nu^*}.$$

SLDA-type functional

$$E_0 = \int \mathcal{E}[n_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}$$

minimization

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From point of view of DFT this step represents uncontrolled approximation,
called *adiabatic approximation*

$$\begin{pmatrix} h_\uparrow(\boldsymbol{r},t) - \mu_\uparrow & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_\downarrow^*(\boldsymbol{r},t) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = i\hbar\frac{\partial}{\partial t} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

DFT method from practical point of view:

DFT method allows for the description of many-body quantum systems with higher accuracy than the mean-field method while keeping the computational complexity at the same level as for the mean-field method.

SLDA-type functional

for ultra-cold atoms

$$E_0 = \int \mathcal{E}[n(\boldsymbol{r}), \tau(\boldsymbol{r}), \boldsymbol{j}(\boldsymbol{r}), \nu(\boldsymbol{r})] d\boldsymbol{r}$$

Dimensionless functional parameters

$$\{A_\lambda, B_\lambda, C_\lambda\}$$

$$\lambda = ak_F$$

Densities

$n(\boldsymbol{r}),\ \tau(\boldsymbol{r}),\ \nu(\boldsymbol{r})$

are defined via

$[u_\eta(\boldsymbol{r}, t), v_\eta(\boldsymbol{r}, t)]^T$

$$\mathcal{E} = \frac{A_\lambda}{2}\left(\tau - \frac{\boldsymbol{j}^2}{n}\right) + \frac{3}{5} B_\lambda\, n\, \varepsilon_F + \frac{C_\lambda}{n^{1/3}} |\nu|^2 + \frac{\boldsymbol{j}^2}{2n}$$

dimensional analysis + symmetries

Kinetic term

Potential term

Pairing term

Center of mass motion

Units:

$\hbar = m = 1$

SLDA-type functional

for ultra-cold atoms

$$E_0 = \int \mathcal{E}[n(\boldsymbol{r}), \tau(\boldsymbol{r}), \boldsymbol{j}(\boldsymbol{r}), \nu(\boldsymbol{r})]d\boldsymbol{r}$$

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dimensional analysis + symmetries

Kinetic term

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Units:

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Example: The simplest choice

BdG (mean-field)

$$A_\lambda \to 1$$

$$B_\lambda \to 0$$

$$C_\lambda \to \frac{4\pi\hbar^2}{(3\pi^2)^{1/3}m}\overbrace{ak_F}^{\lambda}$$

$$\mathcal{E}_{\mathrm{BdG}} = \frac{\tau}{2} + 4\pi a|\nu(\boldsymbol{r})|^2$$

$\downarrow$ *minimization*

$$\begin{pmatrix} -\frac{\hbar^2}{2}\nabla^2 - \mu & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & \frac{\hbar^2}{2}\nabla^2 + \mu \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

$$\Delta = -4\pi a \sum_{|E_n|<E_c} u_n(\mathbf{r})v_n^*(\mathbf{r})\frac{f_\beta(-E_n) - f_\beta(E_n)}{2}$$

There always exists a functional that after minimization provides equations identical to the mean-field equations (zeroth order).

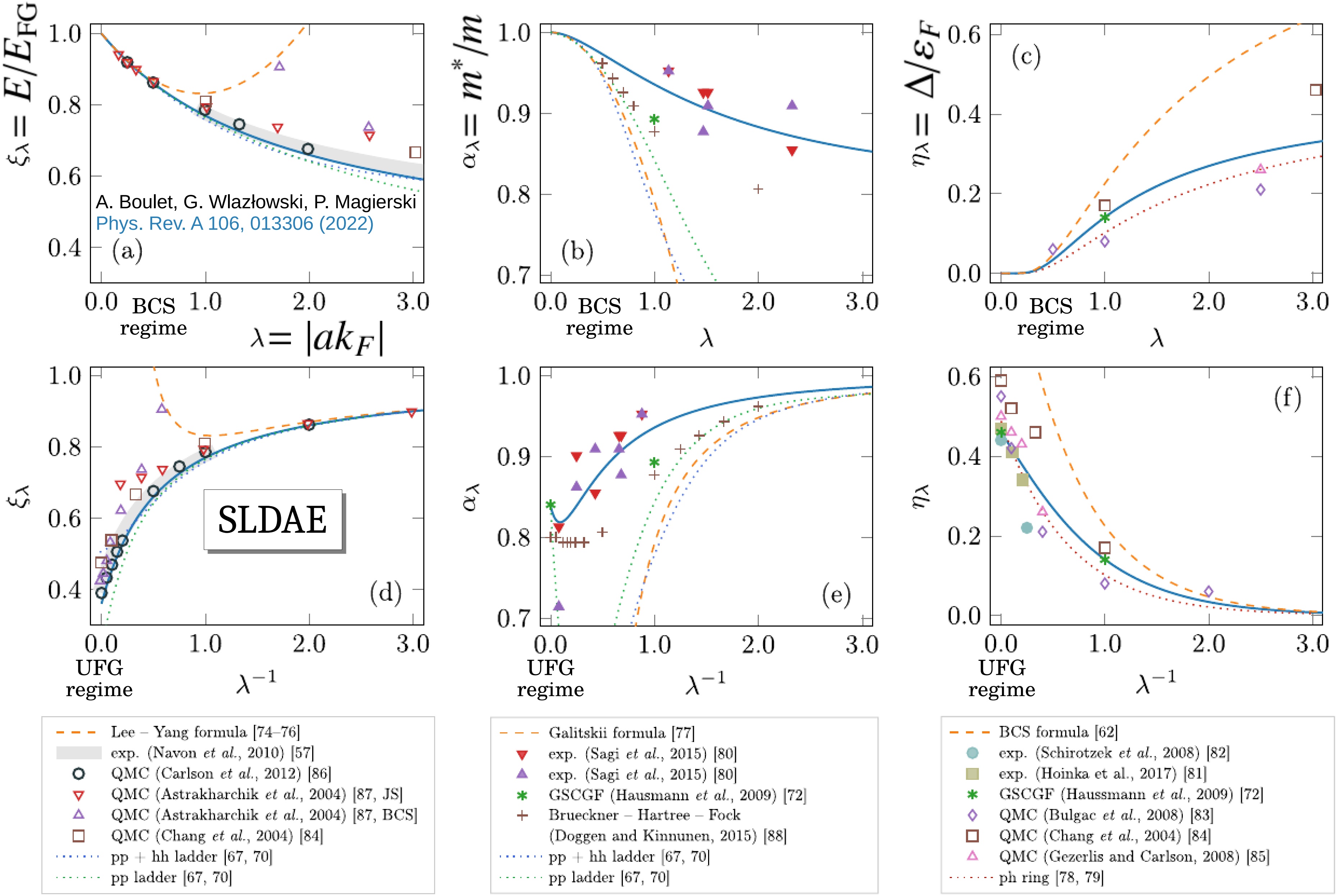
→ *ab initio* cals for E/E_{FG}, Δ/ε_F, m^*/m

→ limiting cases (EFT, scale invariance, ...)

INDUCE

Functional parameters

$\{A_\lambda, B_\lambda, C_\lambda\}$

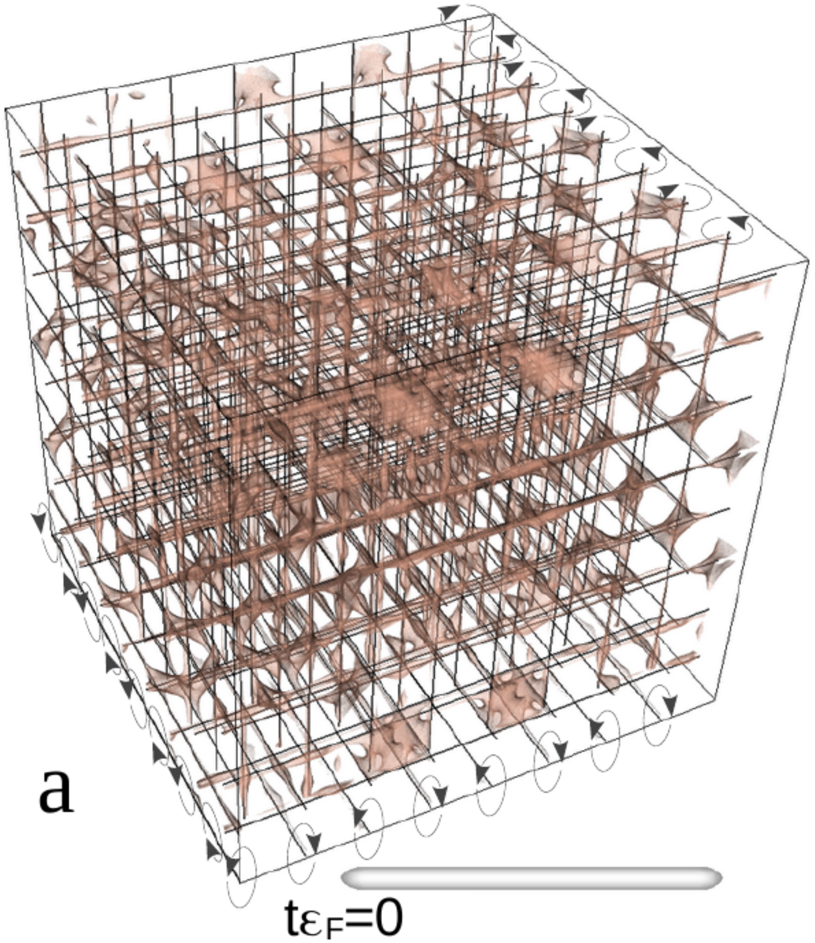


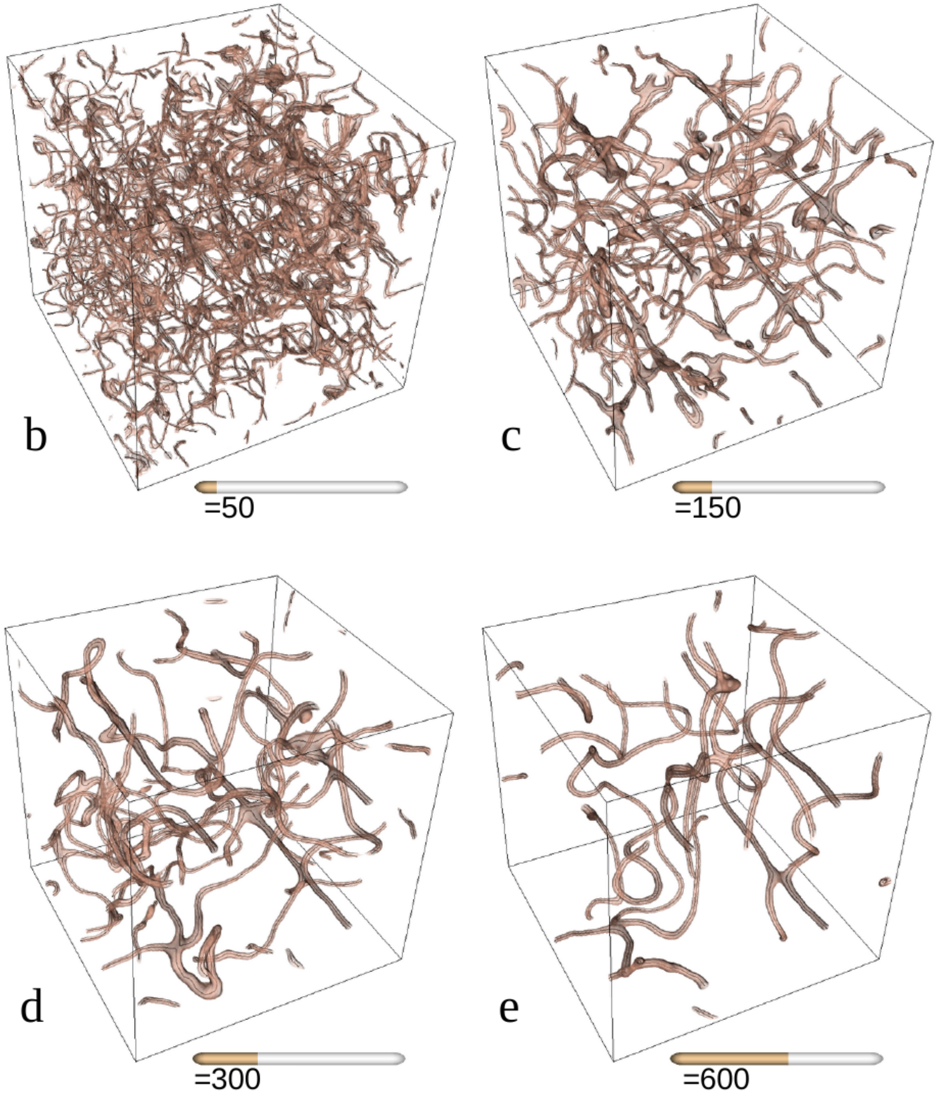
A. Boulet, G. Wlazłowski, P. Magierski

Phys. Rev. A 106, 013306 (2022)

Quantum turbulence in 3D

G. Wlazłowski, M.M.Forbes, S. Sarkar, A. Marek, M. Szpindler,
PNAS Nexus, pgae160 (2024)





initial state:

→ zero temperature (T = 0)
→ regular lattice of imprinted vortices
in all three directions
→ the lattice consists of alternately arranged
vortices and anti-vortices
→ small long-wavelength perturbations of vortex lines

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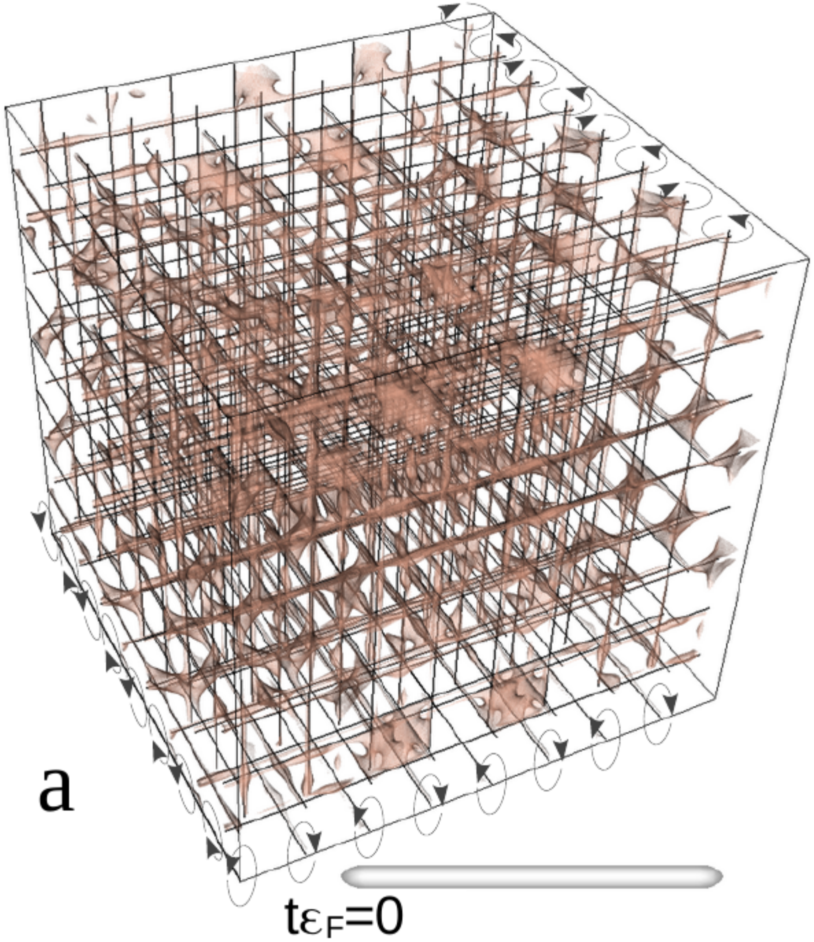
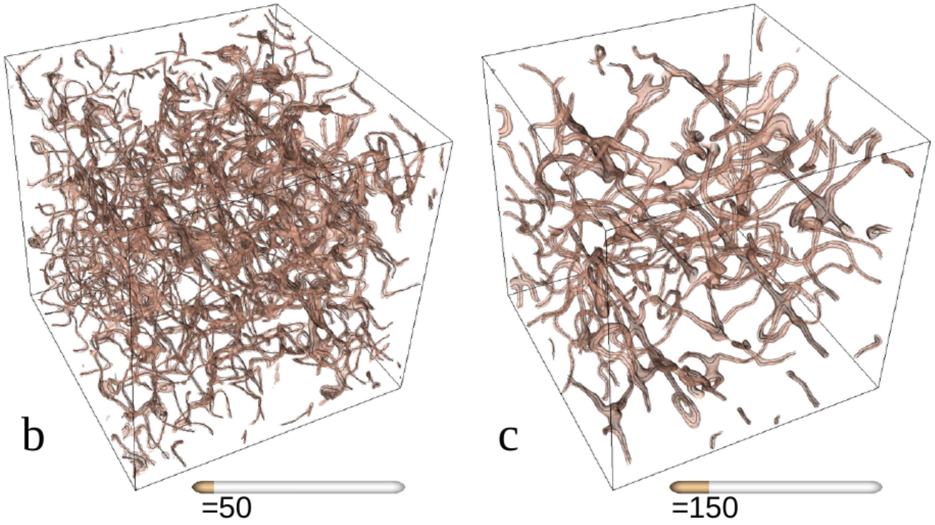




initial state:
- → zero temperature (T = 0)
- → regular lattice of imprinted vortices in all three directions
- → the lattice consists of alternately arranged vortices and anti-vortices
- → small long-wavelength perturbations of vortex lines

Calculations:
- → TDDFT for two coupling constants: $ak_F=\infty$ and ak_F=-1.8
- → modified GPE (Extended Thomas Fermi) for the same initial conditions

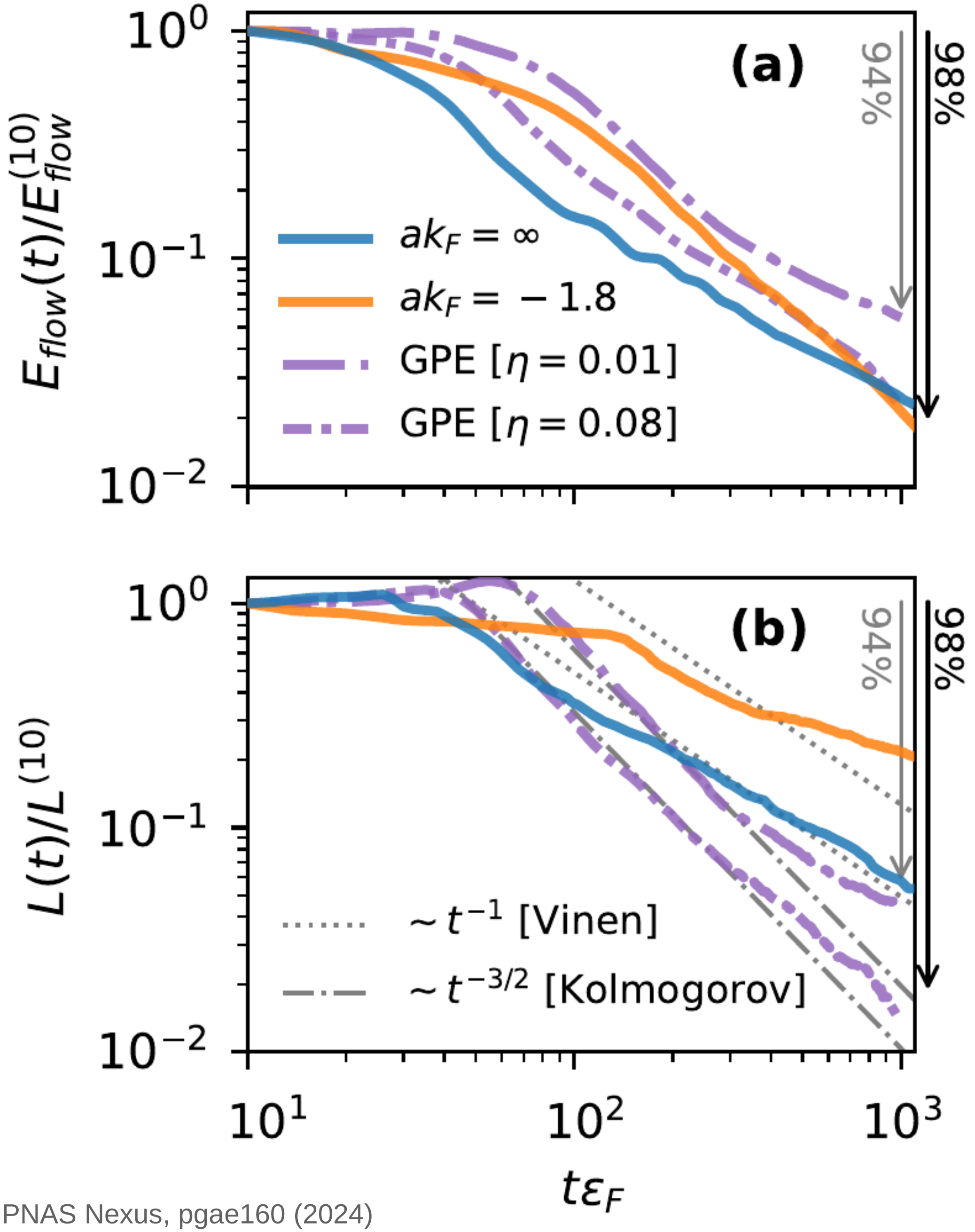
$$i\hbar e^{i\eta}\frac{\partial\psi_B(\boldsymbol{r},t)}{\partial t}=\left(\frac{-\hbar^2\nabla^2}{2m_B}+\mathcal{E}'\big(n_B(\boldsymbol{r},t)\big)\right)\psi_B(\boldsymbol{r},t),$$

phase factor to
model dissipation

mass of dimer
(2m)

effective mean-
field chemical potential
$(=\xi\varepsilon_F)$

Quantum turbulence in 3D – observables

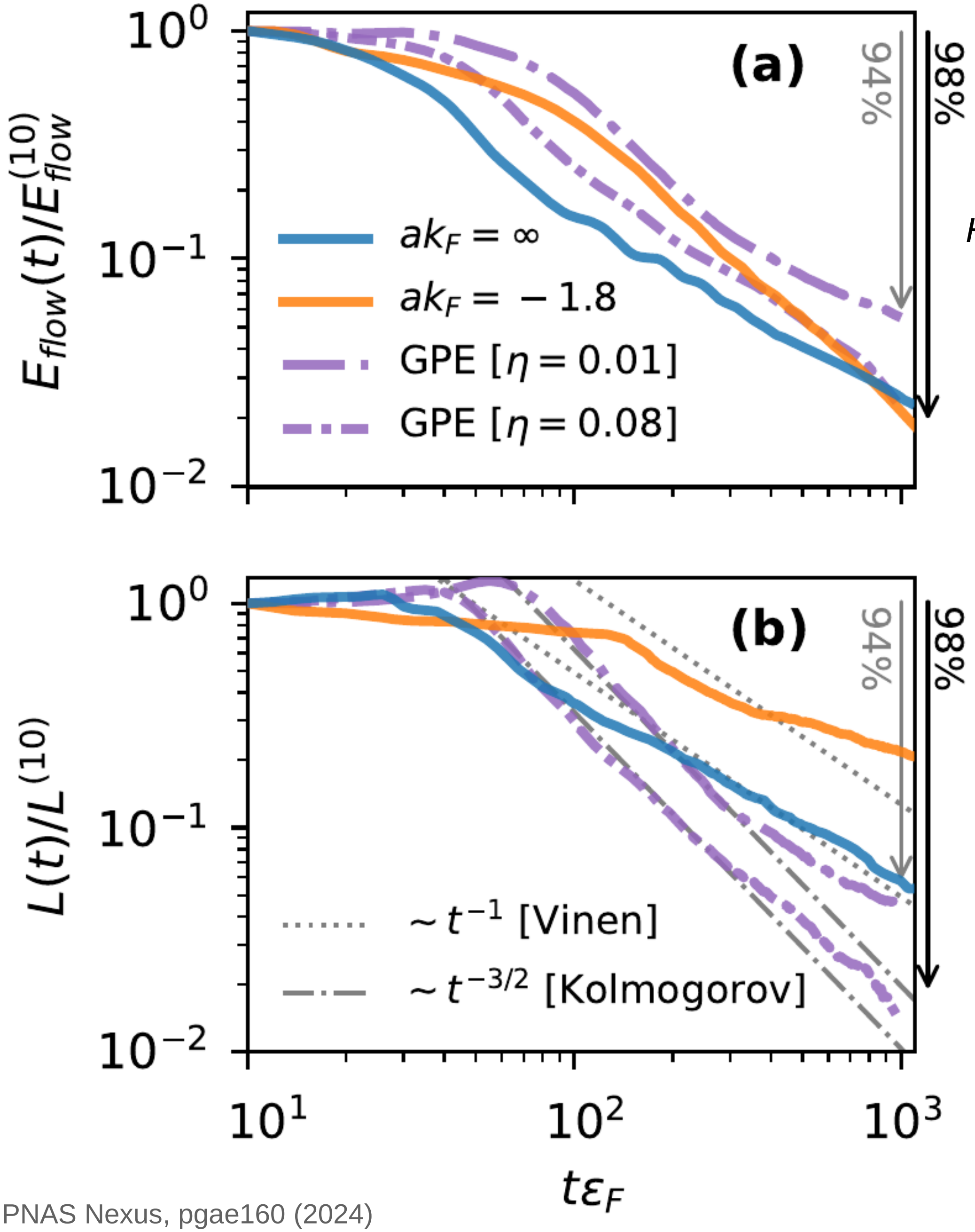


→ flow energy $E_{\mathrm{flow}}(t) = \int \frac{\boldsymbol{j}^2(\boldsymbol{r},t)}{2n(\boldsymbol{r},t)} \mathrm{d}^3\boldsymbol{r},$

→ total vortex length $L(\mathrm{t})$

PNAS Nexus, pgae160 (2024)

Quantum turbulence in 3D – observables



PNAS Nexus, pgae160 (2024)

→ flow energy $E_{\text{flow}}(t) = \int \frac{j^2(\boldsymbol{r},t)}{2n(\boldsymbol{r},t)} \mathrm{d}^3\boldsymbol{r},$

→ total vortex length L(t)

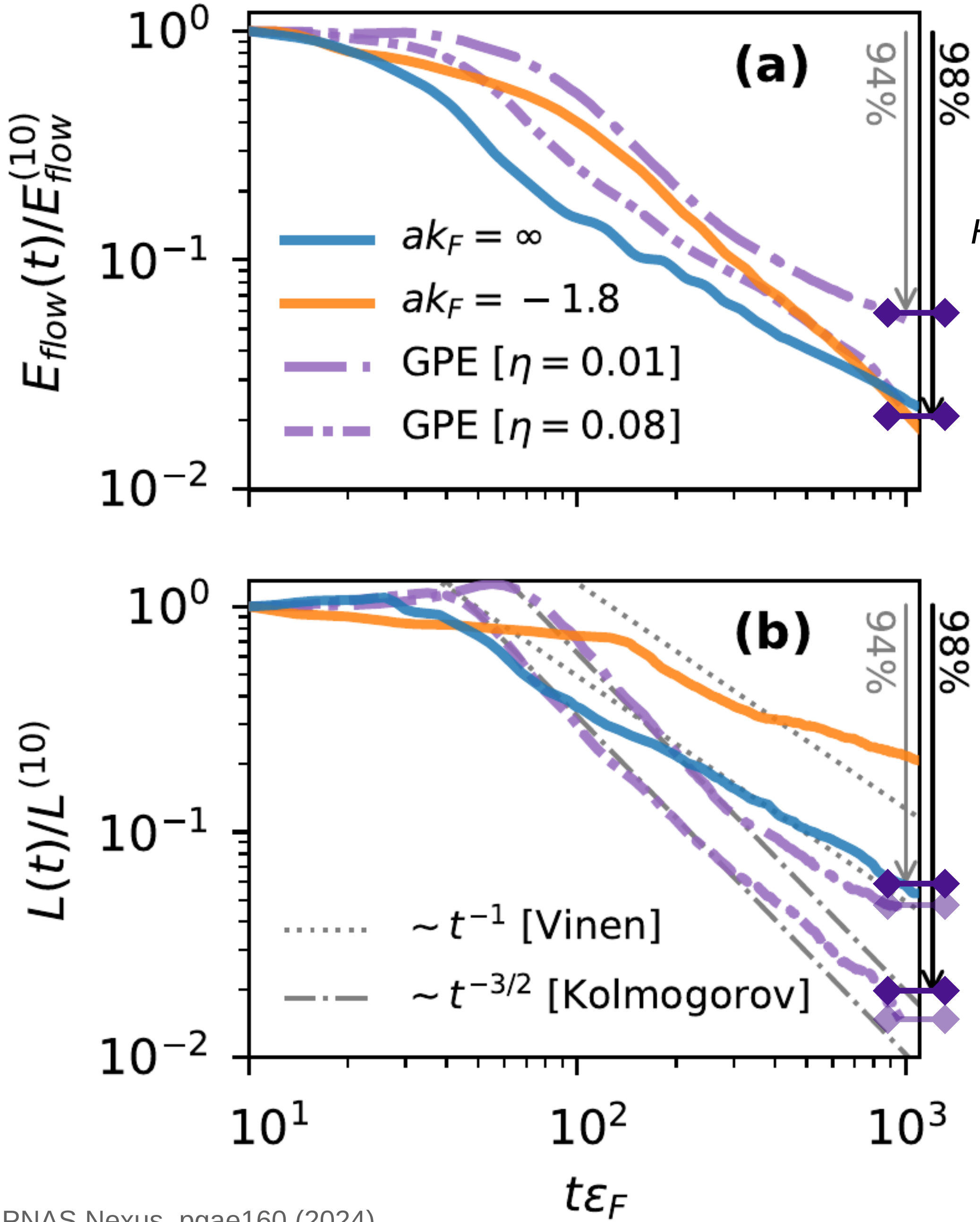
Helmholtz decomposition:

$$E_{\text{flow}} = E_{\text{flow}}^{\text{vortex}} + E_{\text{flow}}^{\text{phonon}}$$

~L(t)

$$E_{\text{flow}} \geqslant E_{\text{flow}}^{\text{vortex}} \Rightarrow \boxed{\frac{E_{\text{flow}}(t)}{E_{\text{flow}}(0)} \geqslant \frac{L(t)}{L(0)}}$$

Quantum turbulence in 3D – observables



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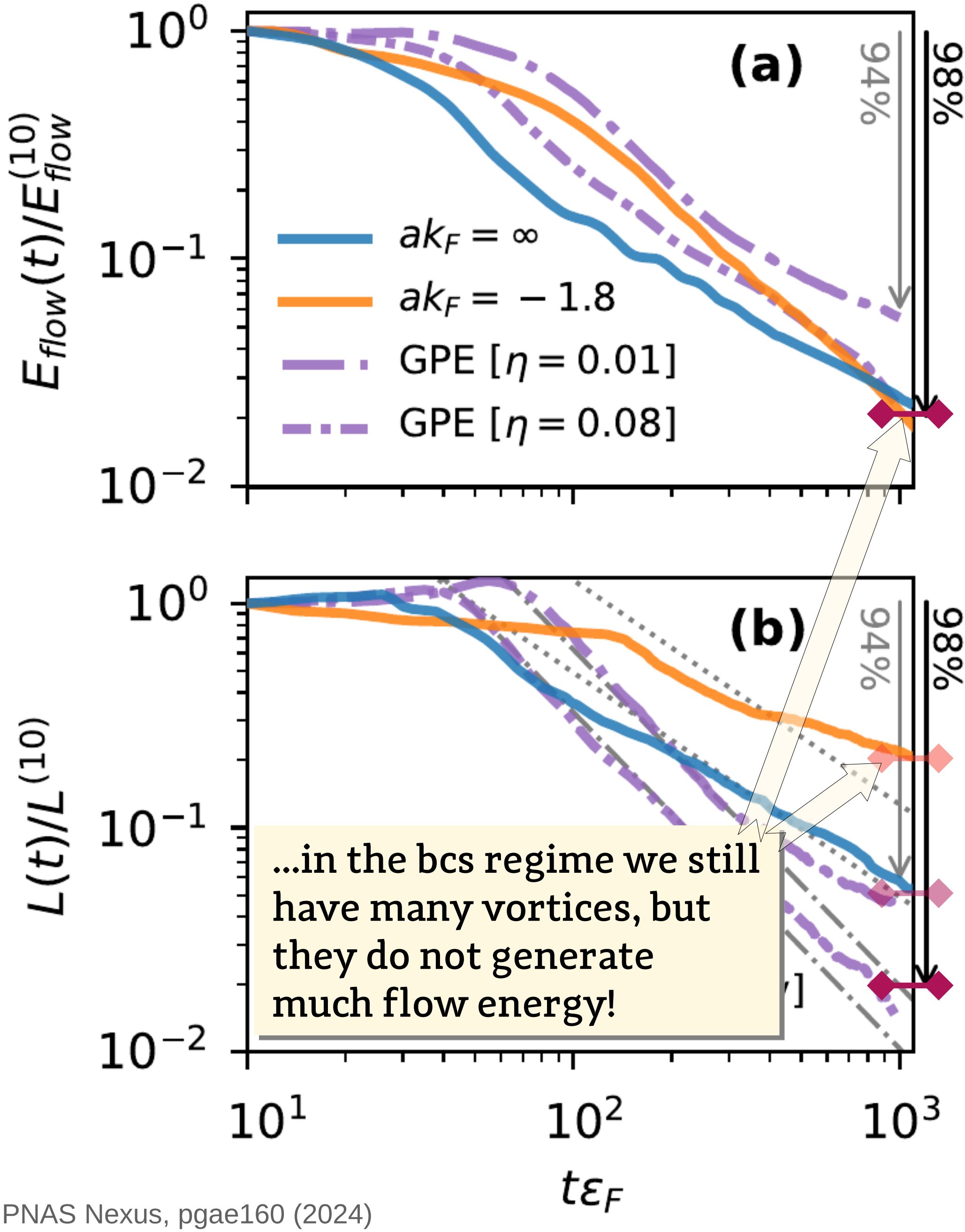
~L(t)

$$E_{\rm flow} \geqslant E_{\rm flow}^{\rm vortex} \Rightarrow \boxed{\frac{E_{\rm flow}(t)}{E_{\rm flow}(0)} \geqslant \frac{L(t)}{L(0)}}$$

→ **The expected result is observed for bosonic (GPE) simulations...**

PNAS Nexus, pgae160 (2024)

Quantum turbulence in 3D – observables



...in the bcs regime we still have many vortices, but they do not generate much flow energy!

PNAS Nexus, pgae160 (2024)

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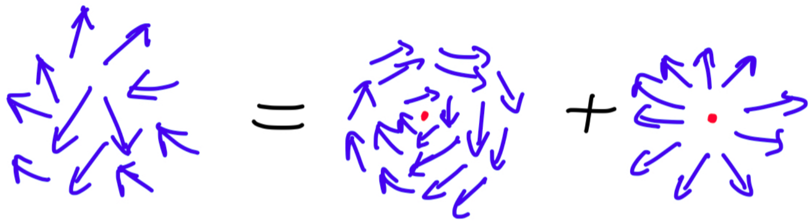


$$E_{\text{flow}} = E_{\text{flow}}^{\text{vortex}} + E_{\text{flow}}^{\text{phonon}}$$

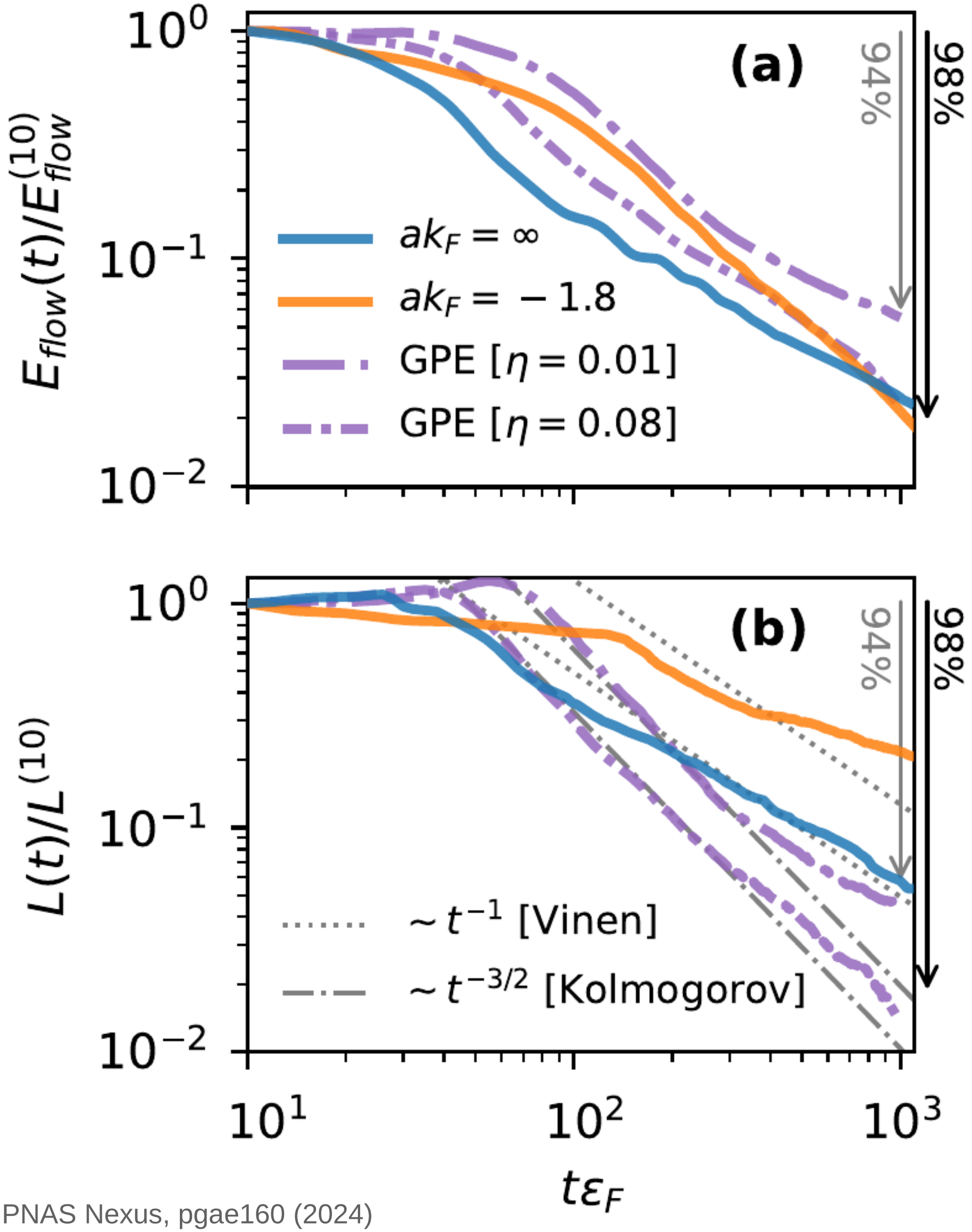
~L(t)

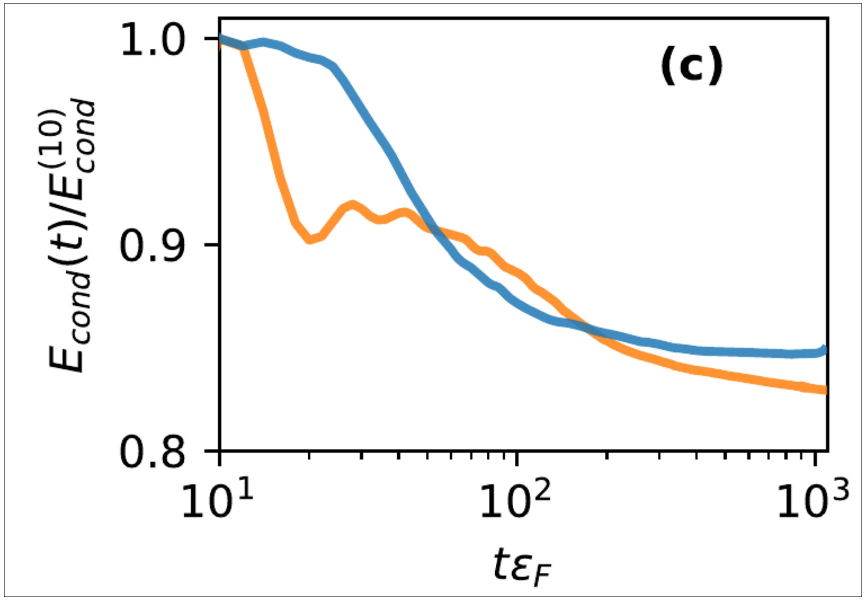
$$E_{\text{flow}} \geqslant E_{\text{flow}}^{\text{vortex}} \Rightarrow \boxed{\frac{E_{\text{flow}}(t)}{E_{\text{flow}}(0)} \geqslant \frac{L(t)}{L(0)}}$$

→ **The expected result is observed for bosonic (GPE) simulations...**

→ **but for fermionic simulations (TDDFT) there is qualitatively different behavoir!**

Quantum turbulence in 3D – observables



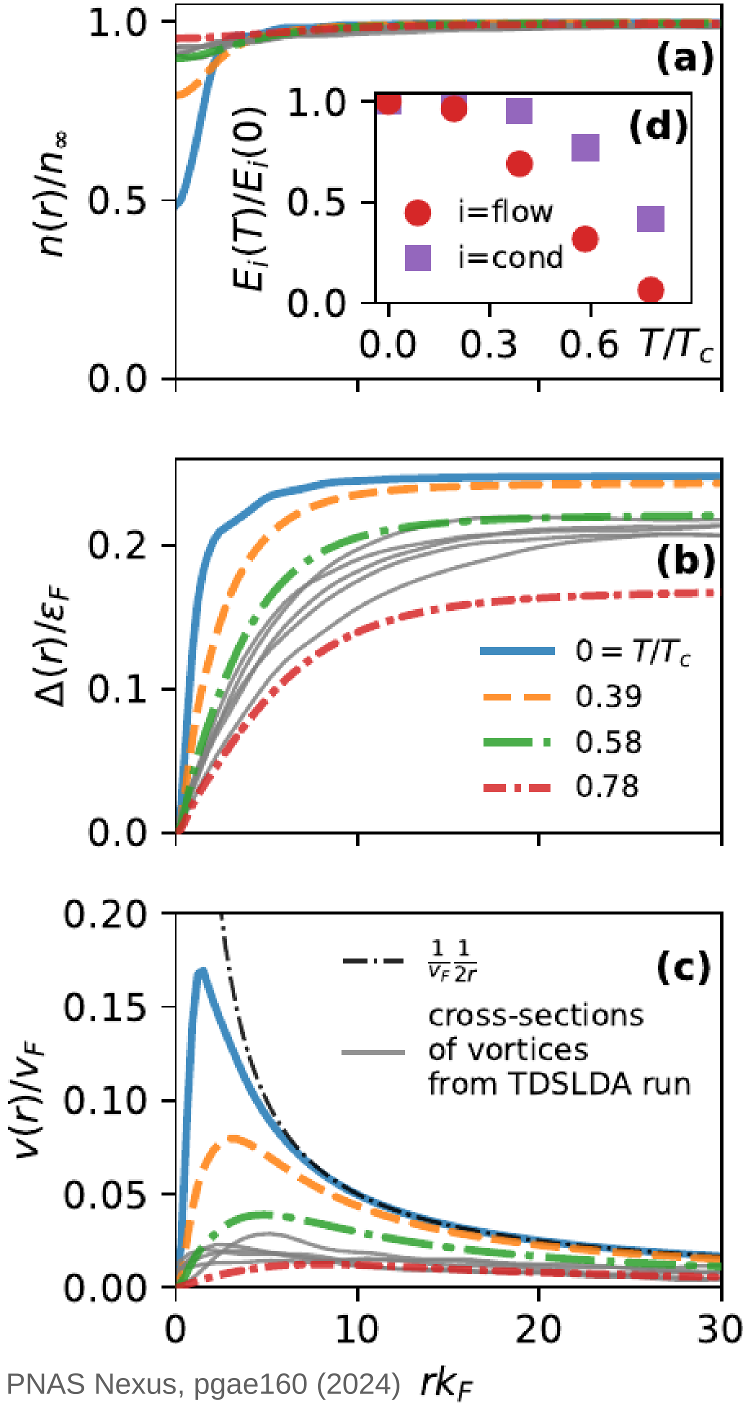


$$E_{\rm BCS} = E_{\rm FG} - \frac{3|\Delta|^2}{8\varepsilon_F}N$$

$$E_{\rm cond} = \int \frac{3}{8}\frac{|\Delta(\mathbf{r})|^2}{\varepsilon_F(\mathbf{r})}n(\mathbf{r})d\mathbf{r}$$

… we observed that the Cooper pairs condensate is depleted during the evolution... → **pair breaking!**

PNAS Nexus, pgae160 (2024)



Radial dependence of the:

(a) density n(r),

(b) order parameter Δ(r),

(c) velocity v(r) = j(r)/n(r)

for a single straight vortex line at various temperatures in the BCS regime ($k_F a = -1.8$).

PNAS Nexus, pgae160 (2024)

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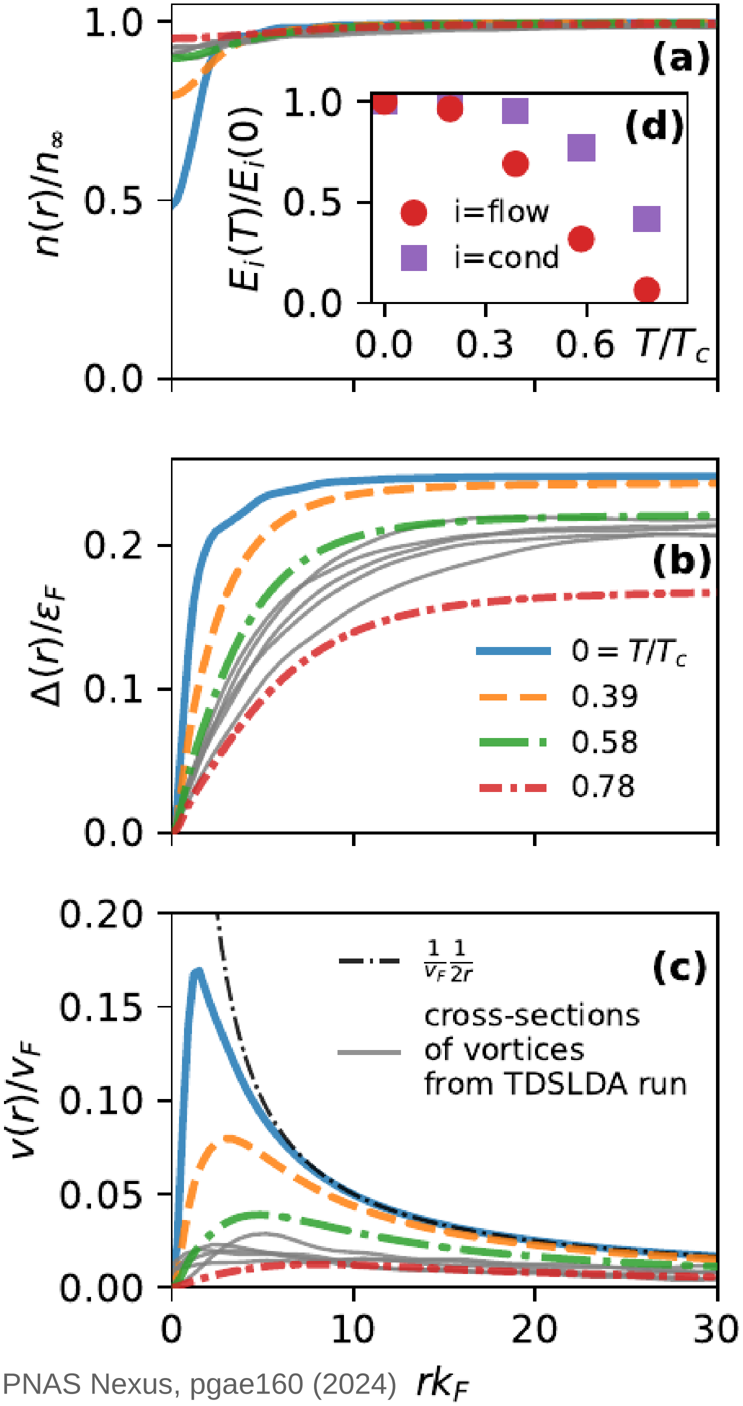
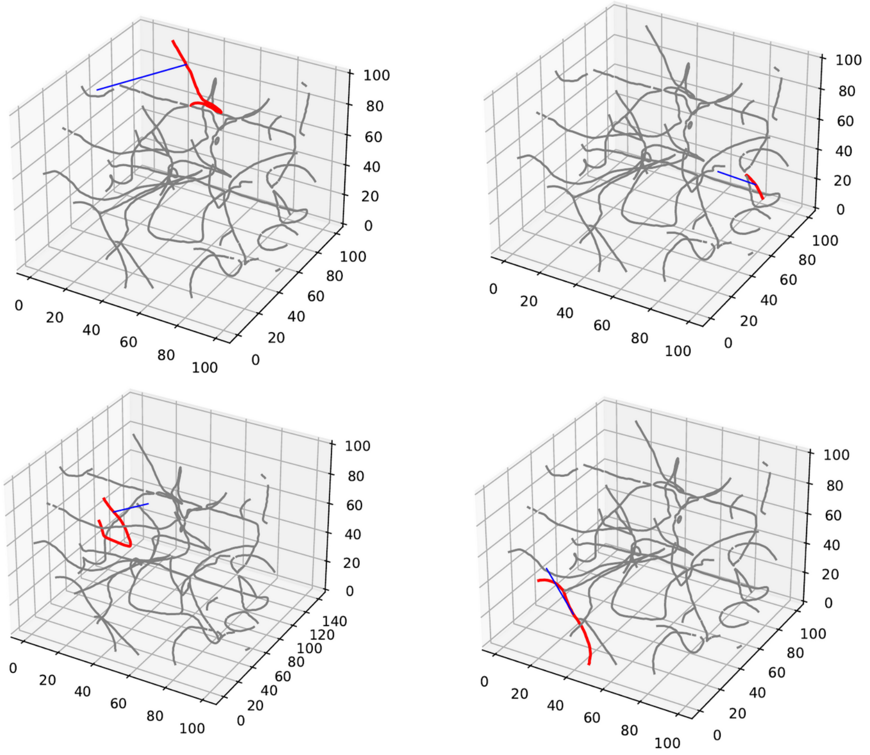
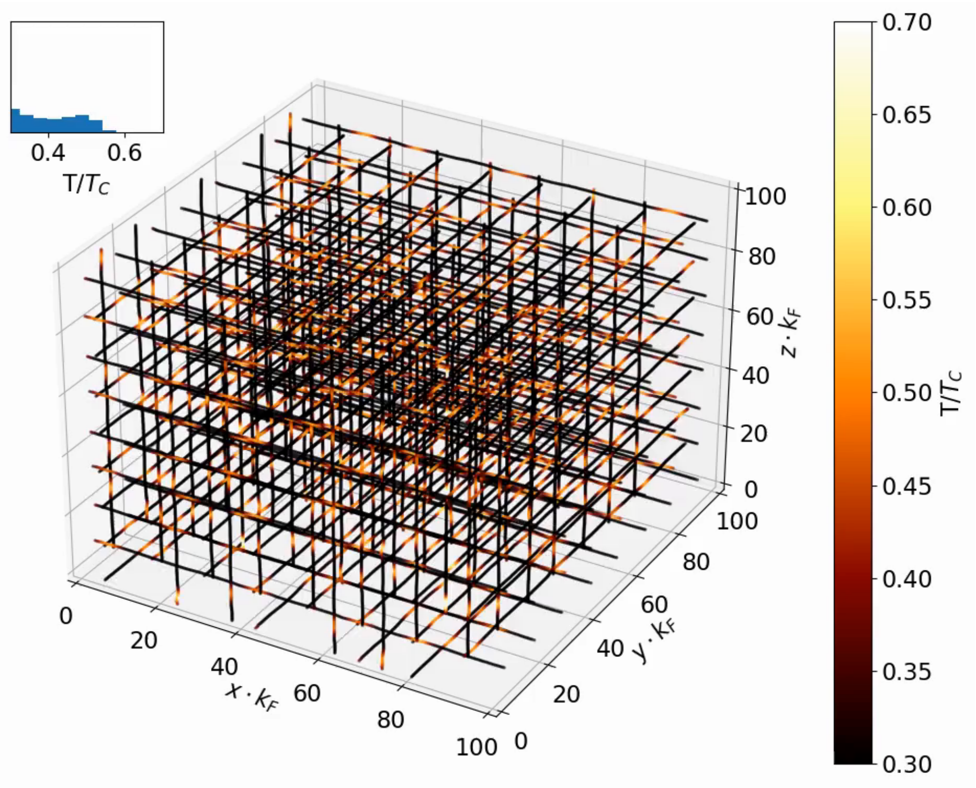


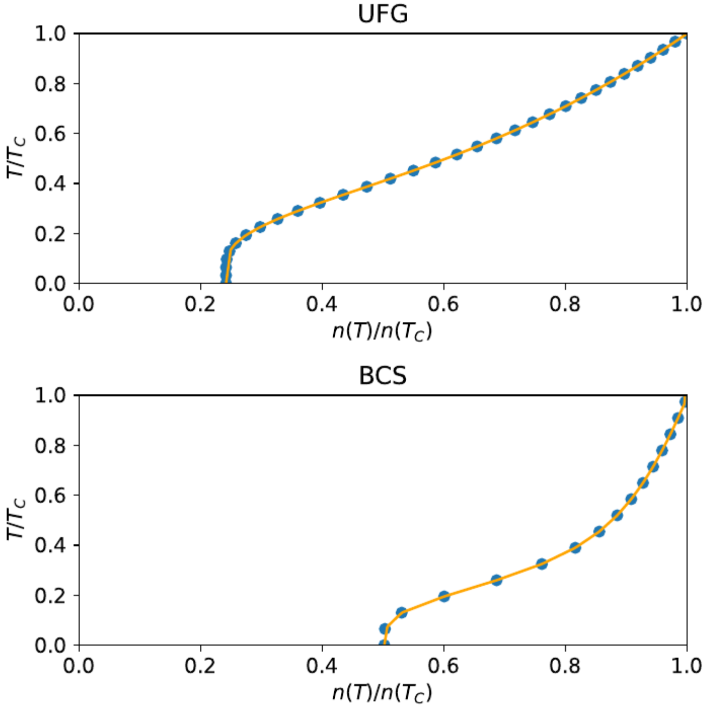
The thin gray lines show the profiles of selected vortices from the TDDFT calcs taken at time $t\varepsilon_F$ = 1,000
→ ***the system effectively heats up!***



PNAS Nexus, pgae160 (2024)

The temperature dependence of the vortex-core density n_{core} allows use fermionic vortices as a local thermometers.





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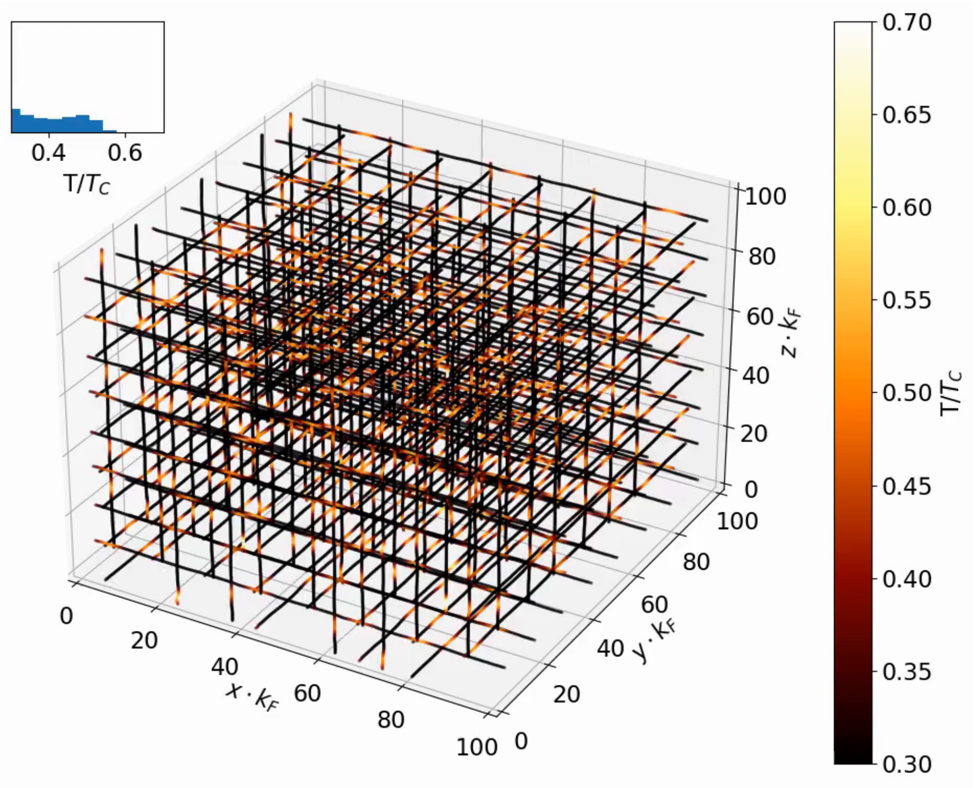
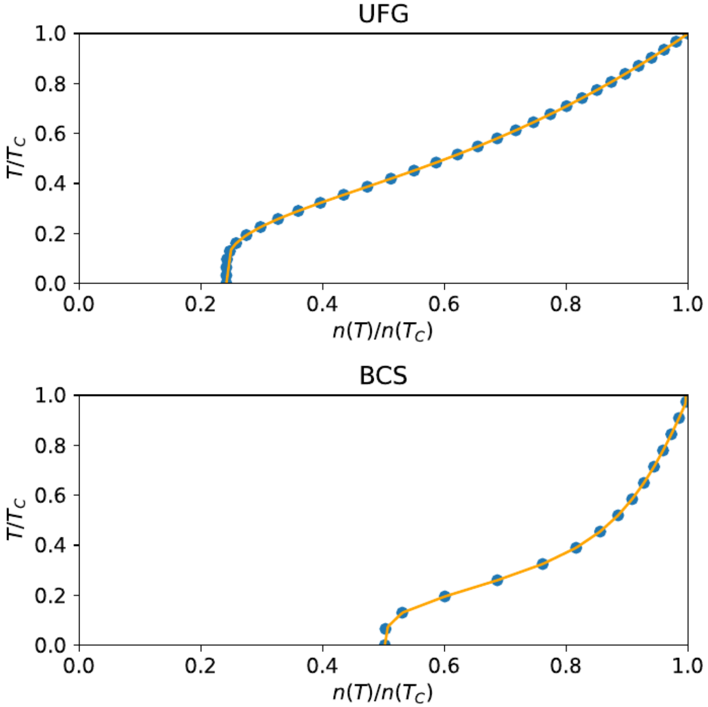




→ the effective temperature of vortex lines is higher in regions of higher curvature (reconnections, kelvin waves)

→ similarity to the heating of wire, which is sharply bent back and forth...

→ … also to mechanism proposed by Silaev.

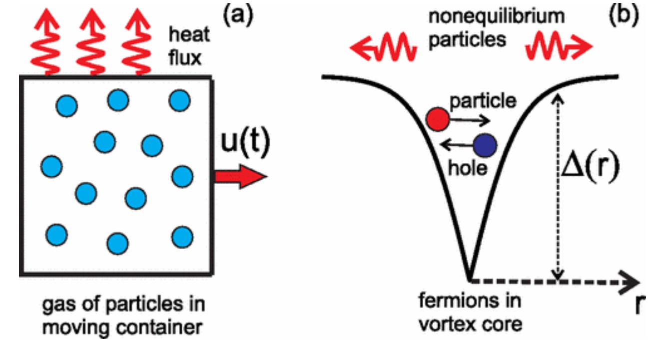
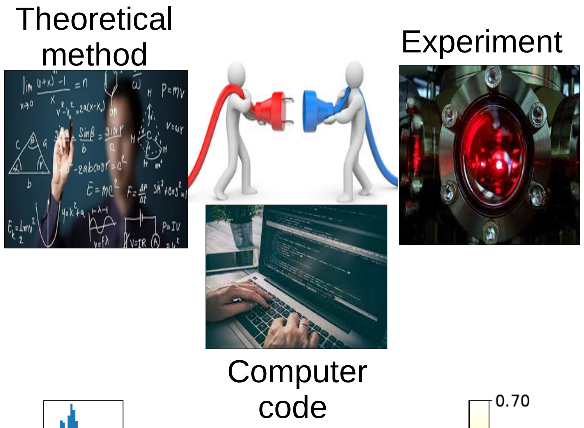


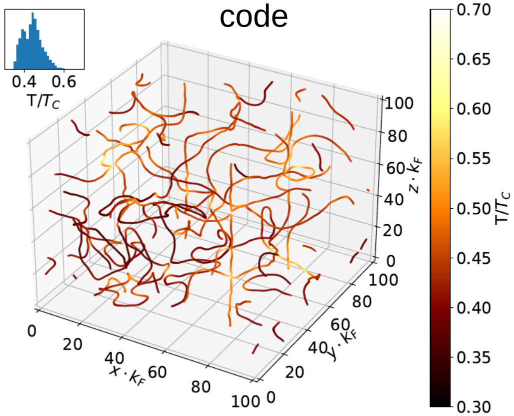
Fig. from: M.A. Silaev, *Universal Mechanism of Dissipation in Fermi Superfluids at Ultralow Temperatures,* Phys. Rev. Lett. 108, 045303 (2012)

PNAS Nexus, pgae160 (2024)

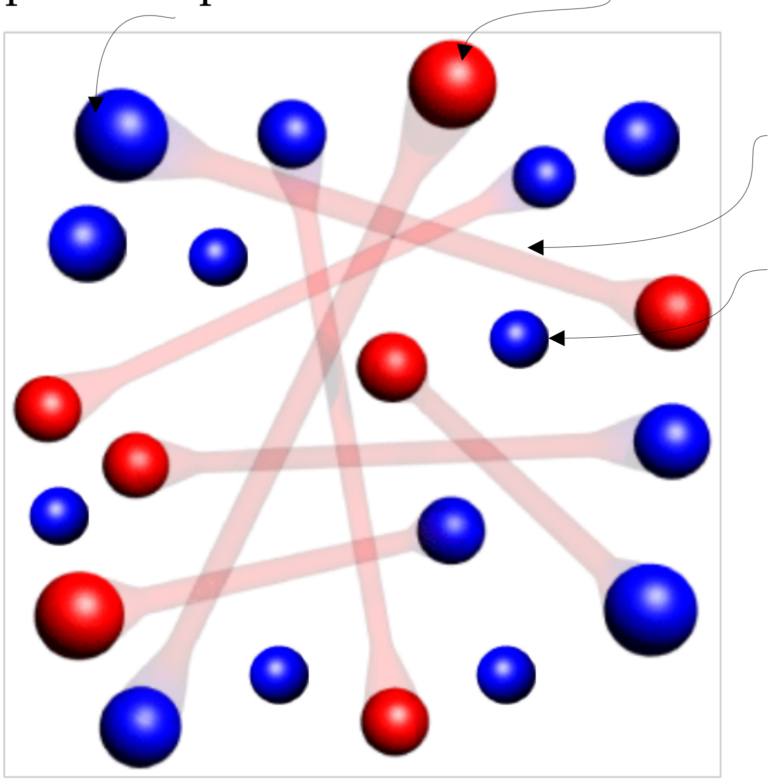
SUMMARY (1)

- (TD)DFT is general purpose framework: it overcomes limitations of mean-field approach, while keeping numerical cost at the same level as (TD)BdG calculations.
- (TD)DFT, its implementations and HPC reached the level of maturity that allows for providing predictions for large and complex systems: ~10^4-10^5 atoms.
- Dissipation mechanisms play a key role in differentiating fermionic from bosonic turbulence:
  - → *role of pair breaking mechanism (production of the "normal component") increases as we move towards BCS regime!*





Superfluidity in spin-imbalanced systems ($N_{\uparrow} \neq N_{\downarrow}$)



Spin-down particle

Spin-up particle

Cooper pairs
~superfluid component (?)

unpaired particles
~normal component (?)

Superfluidity in spin-imbalanced systems ($N_\uparrow \neq N_\downarrow$)

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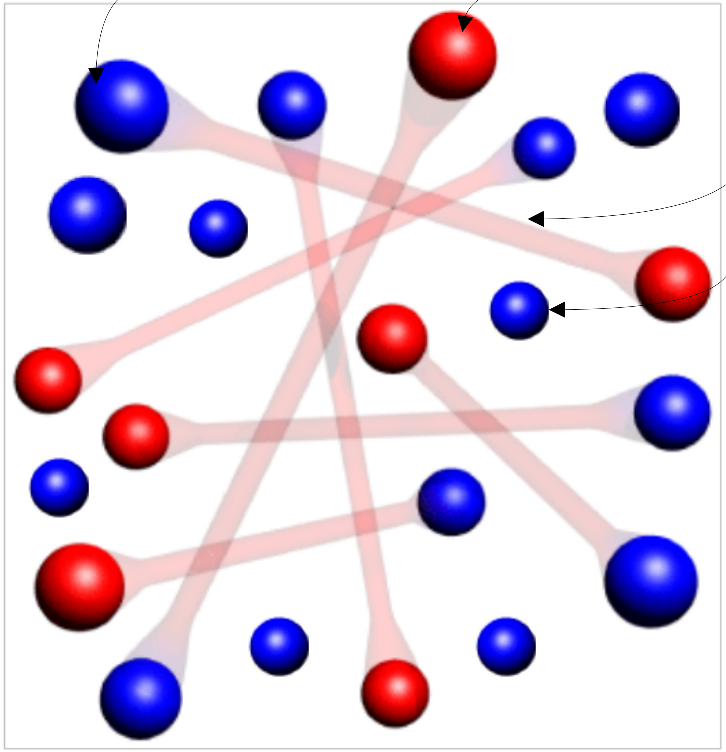
Spin-up particle



Cooper pairs
~superfluid component (?)

unpaired particles
~normal component (?)

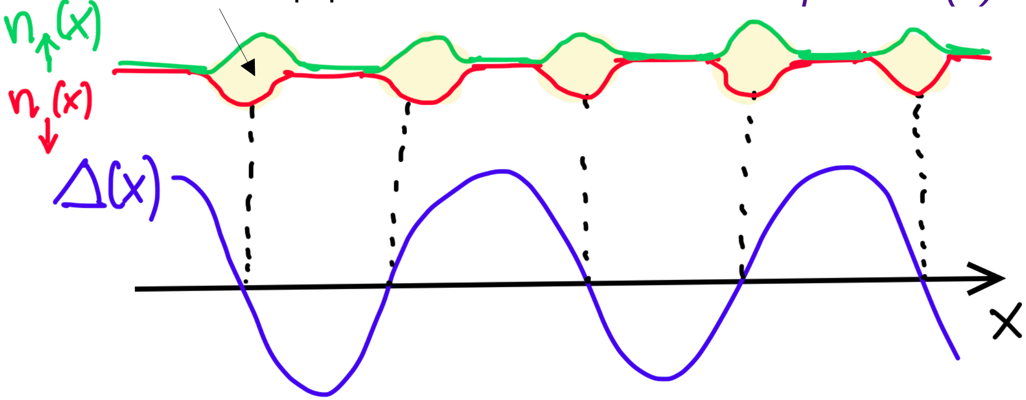
Larkin and Ovchinnikov (LO), and Fulde and Ferrel (FF) proposed new state, with spatial modulation of the pairing field Δ (due to the mismatch between the Fermi surfaces).

$$\Delta(\mathbf{r}) \sim |\Delta| \cos(\mathbf{q}\cdot\mathbf{r})$$

This so-called LOFF phase remains one of the long-sought phenomena in ultracold Fermi gases
… recent review: J.J. Kinnunen et al 2018
Rep. Prog. Phys. 81 046401

unpaired particles
accumulates in places
where |Δ|≈0

Non-uniformly distributed
normal component (?)



Superfluidity in spin-imbalanced systems from numerical modeling

Instead of postulating ansatz for the pairing field [like $\Delta(x)=|\Delta(x)|\cos(kx)$], we let a computer to search for the energy minimum:
→ *B. Tüzemen et al 2023 New J. Phys. 25 033013*

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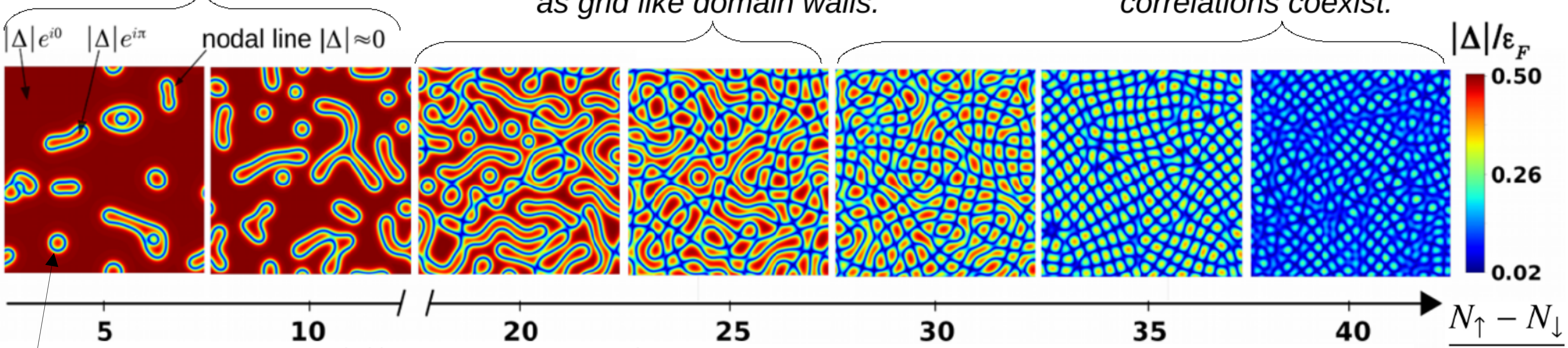
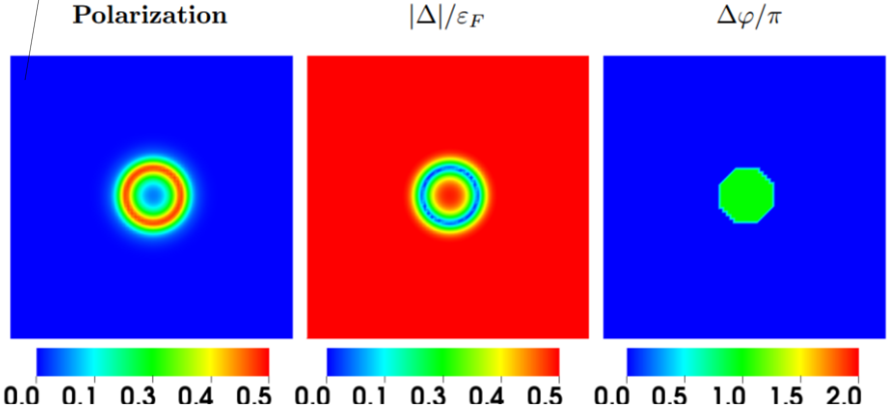
At low polarization values we predict the structure of the system as consisting of several spin-polarized droplets.

As the polarization increases, the system self-organizes into a ***disordered structures*** *similar to liquid crystals, and energetically they can compete with ordered structures such as grid like domain walls.*

At higher polarizations the system starts to develop regularities that, in principle, can be called supersolid, where periodic density modulation and pairing correlations coexist.





B. Tüzemen et al, Acta Phys. Pol. B 51, 595 (2020)

Superfluidity in spin-imbalanced systems from numerical modeling

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But general concept that we have non-uniformly distributed normal component still holds...

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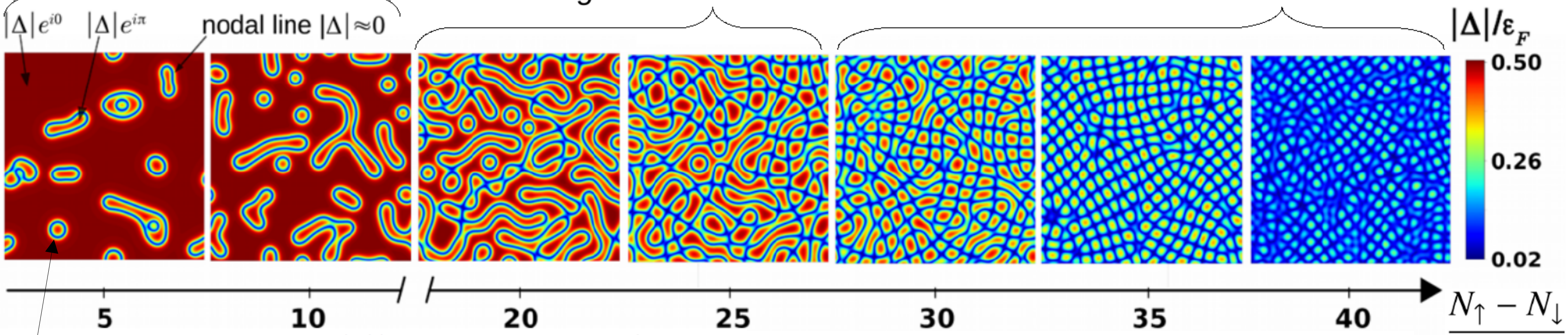
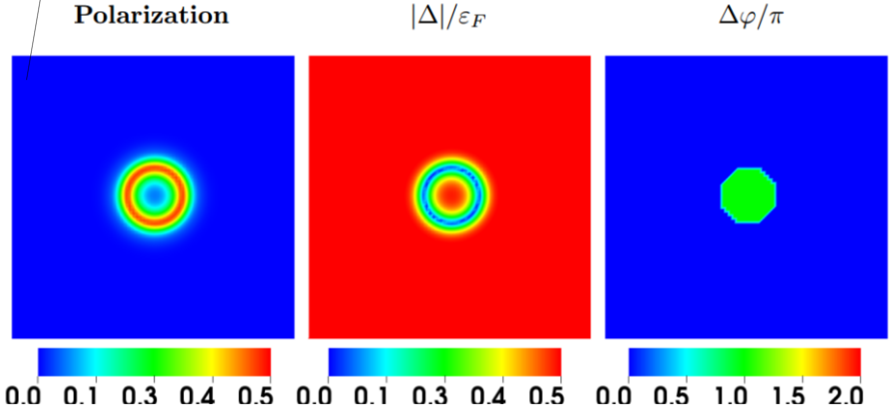
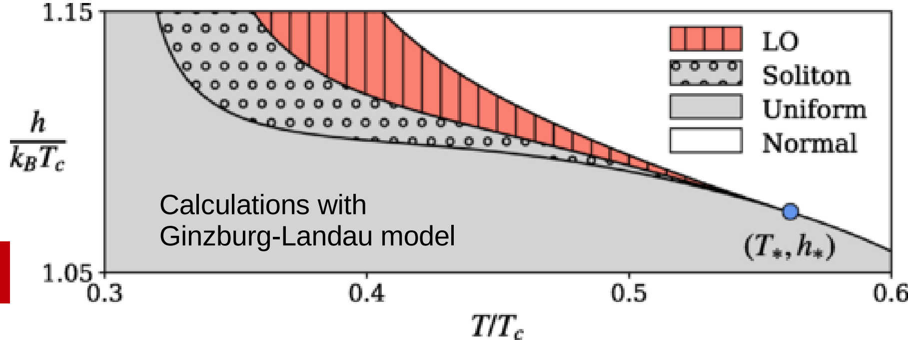




B. Tüzemen et al, Acta Phys. Pol. B 51, 595 (2020)

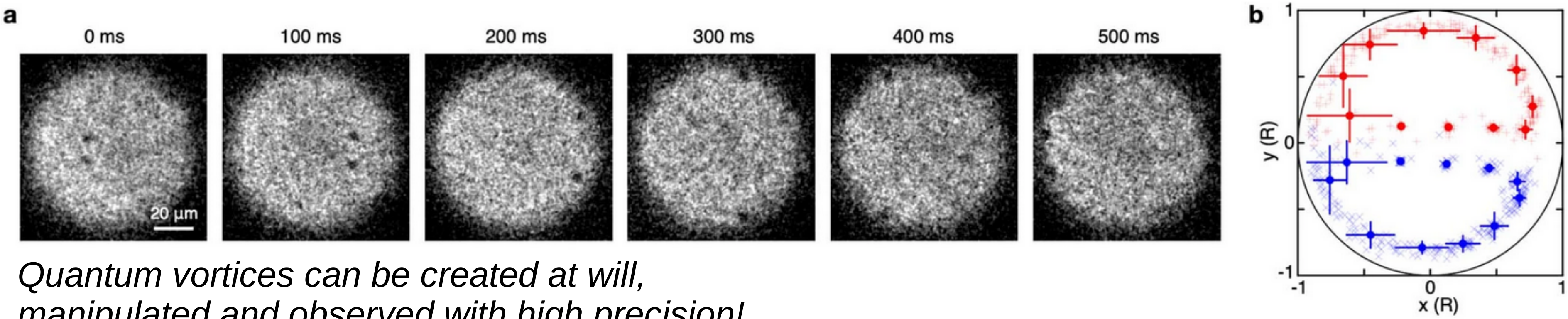
Ring solitons and soliton sacks in imbalanced fermionic systems by M. Barkman, et al, Phys. Rev. Research 2, 043282 (2020)



Calculations with Ginzburg-Landau model

Quantum vortices as a probe of the medium

Inspired by LENS experiments with ^{6}Li atoms (G. Roati's group)
Figures below from: W. J. Kwon, et.al., Nature 600, 64-69 (2021)



Quantum vortices can be created at will,
manipulated and observed with high precision!

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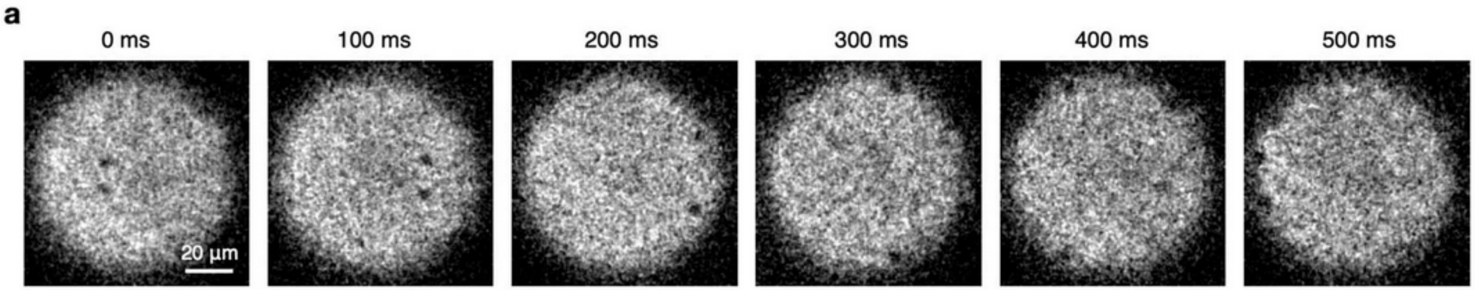
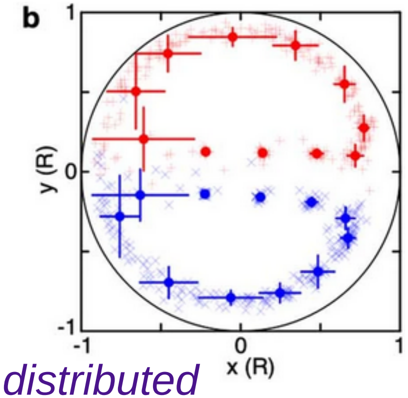




Quantum vortices can be created at will, manipulated and observed with high precision!

normal component distributed
uniformly *non-uniformly*

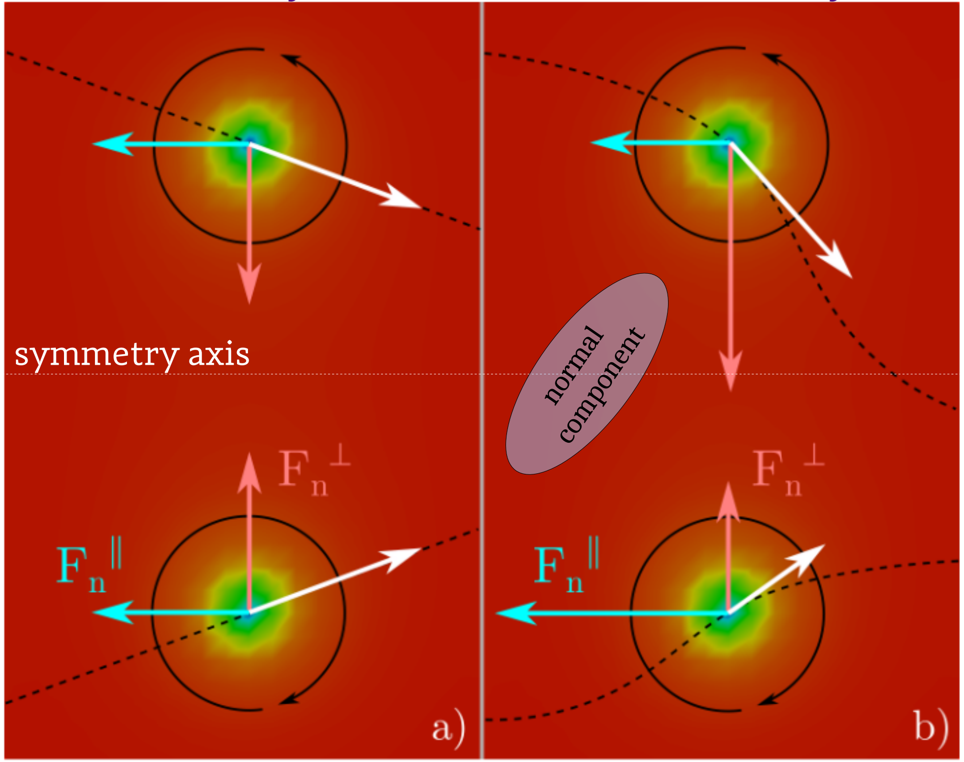
Vortex point model: $\boldsymbol{F}_M + \boldsymbol{F}_N = 0$

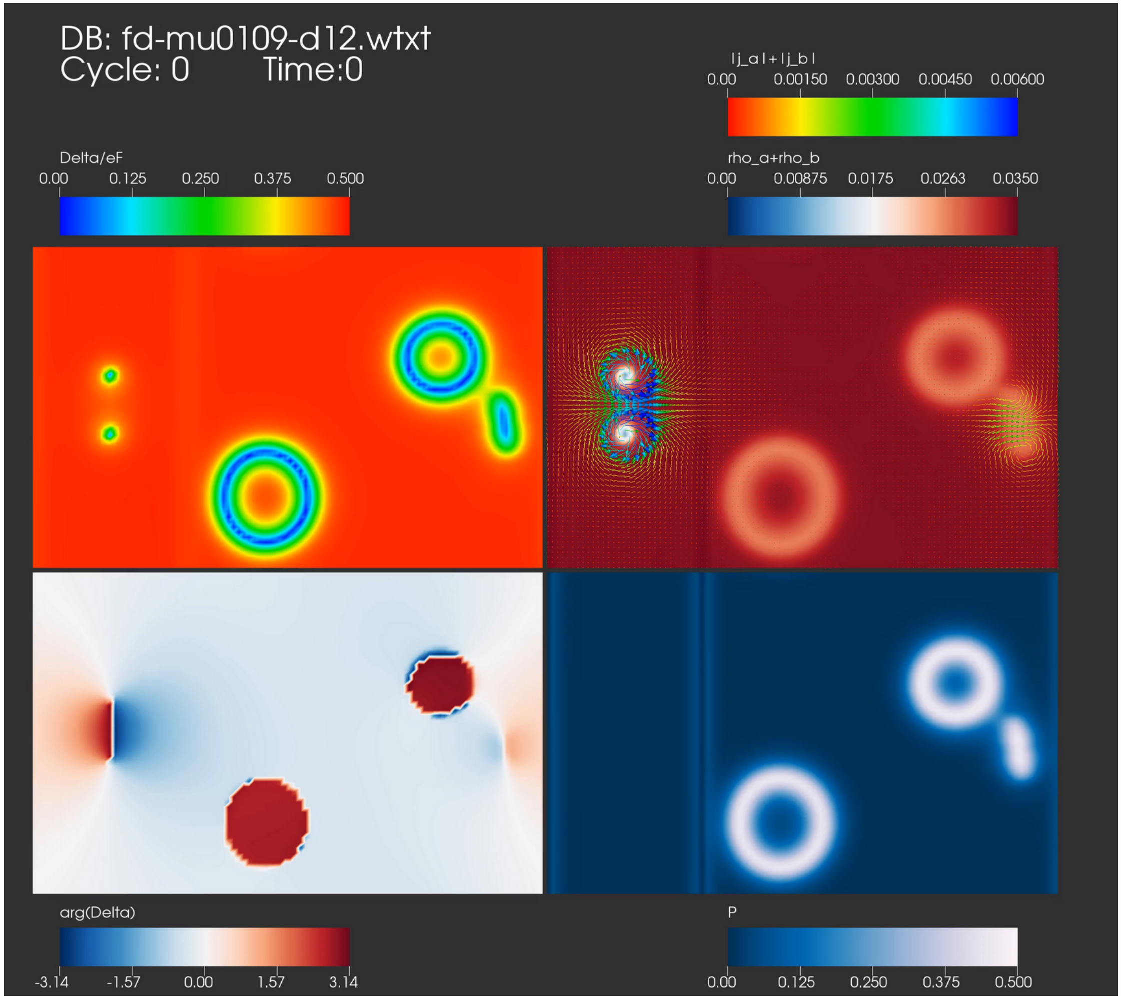
$$\boldsymbol{F}_M = \rho_s \kappa \hat{\boldsymbol{z}} \times (\boldsymbol{v}_i - \boldsymbol{v}_s^{(j \neq i)})$$

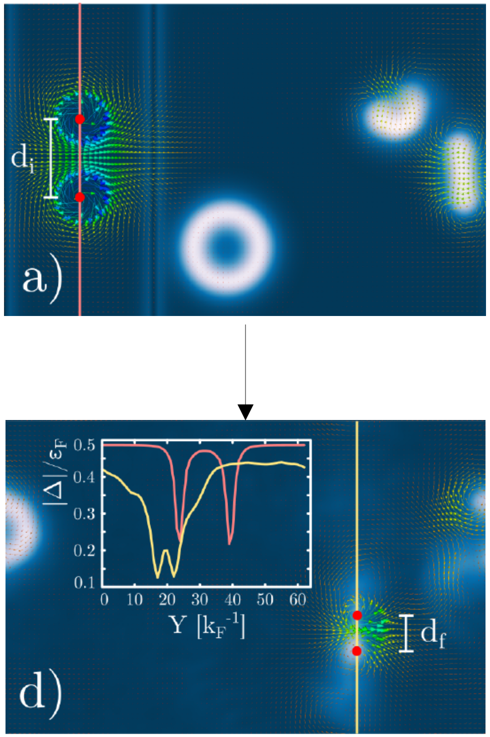
Magnus force

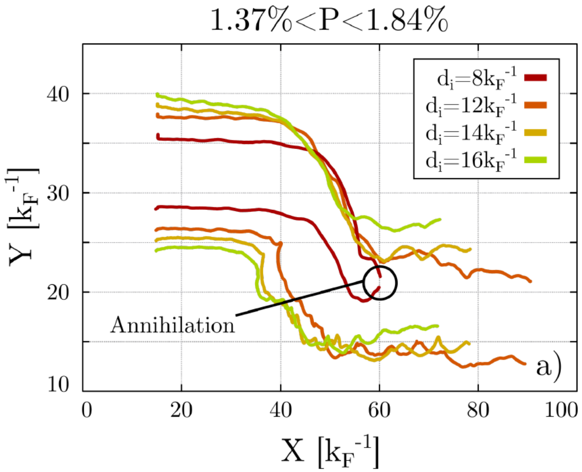
$$\boldsymbol{F}_N = D(\boldsymbol{v}_i - \boldsymbol{v}_n) + D' \hat{\boldsymbol{z}} \times (\boldsymbol{v}_i - \boldsymbol{v}_n)$$

Dissipative forces:
sensitive to presence of the normal component!



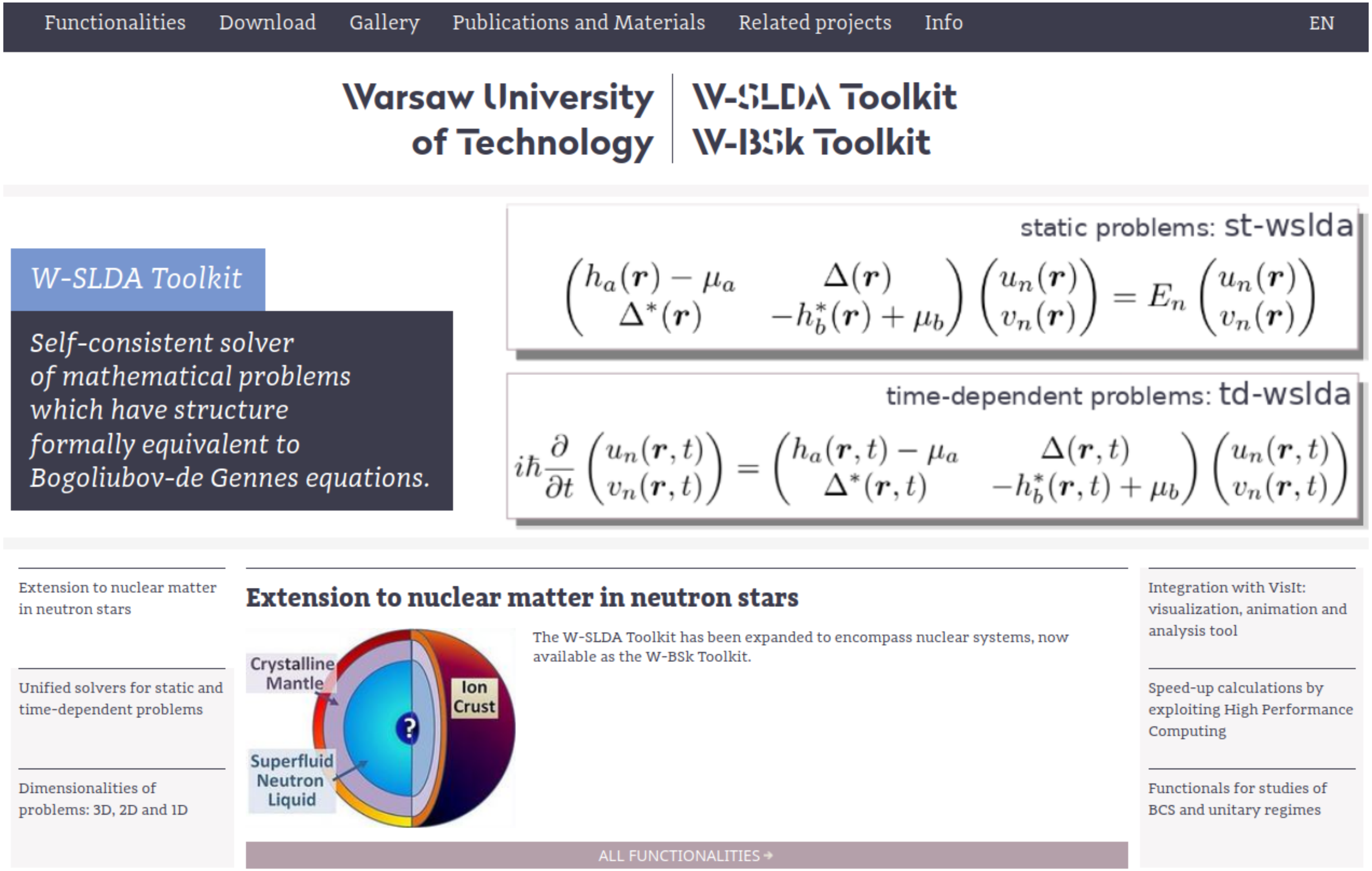






TDDFT result:
Vortex dipoles can be used as robust probes if the normal component is distributed uniformly or non-uniformly.

In preparation: A. Barresi, P. Magierski, G. Wlazłowski (2024)



Functionalities Download Gallery Publications and Materials Related projects Info EN

Warsaw University of Technology | W-SLDA Toolkit
W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r})-\mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r})+\mu_b \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t)-\mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t)+\mu_b \end{pmatrix}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars

The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

Speed-up calculations by exploiting High Performance Computing

Functionals for studies of BCS and unitary regimes

can run on “small” computing clusters as well as leadership supercomputers
(depending on the problem size)

High
Performance
Computing

NVIDIA CUDA

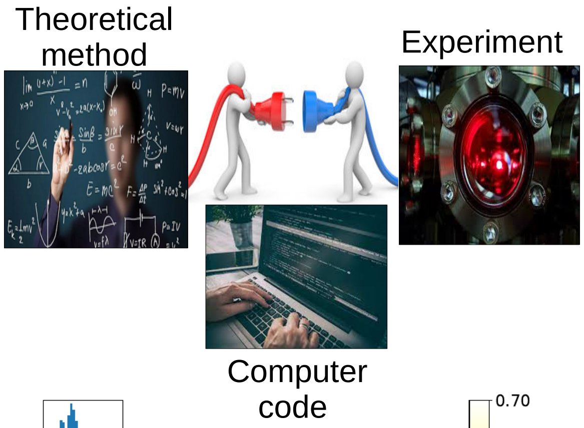
AMD ROCm

open source

... all tools we create for Fermi gas simulations are publicly accessible as open-source...

http://wslda.fizyka.pw.edu.pl/

SUMMARY (2)



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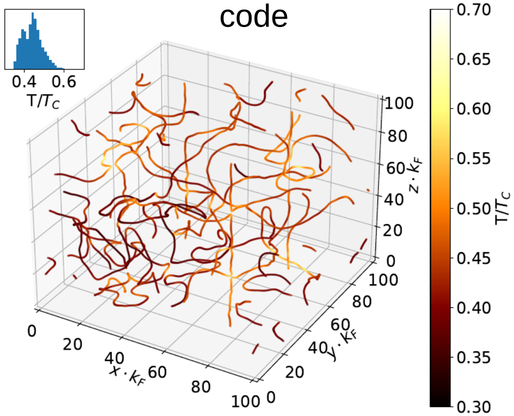
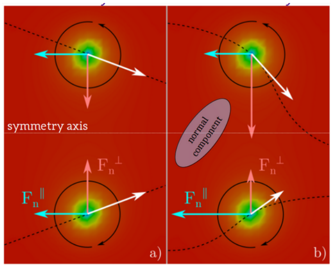
  - → *role of pair breaking mechanism (production of the "normal component") increases as we move towards BCS regime!*
- *Quantum vortices can be used as probes if the normal component is distributed uniformly or non-uniformly.*
  - → *indirect tests for existence of exotic phases in spin-imbalanced gases.*





Thank you!

Contributors: P. Magierski, A. Barresi (WUT); A. Boulet (WUT→Le Mans); M. Forbes, S. Sarkar, (WSU); A. Bulgac (UW); B. Tüzemen (WUT→PAS); T. Zawiślak (WUT→U.Trento); A. Marek (MPCDF), M. Szpindler (Cyfronet).

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