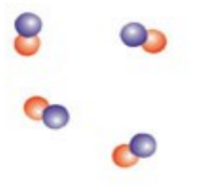
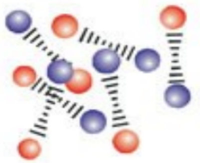
Towards accurate modeling of neutron star crust properties and what we can learn from them about the core

Gabriel Wlazłowski

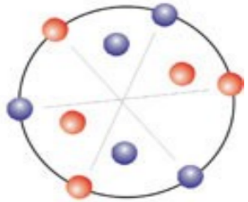
Warsaw University of Technology
University of Washington



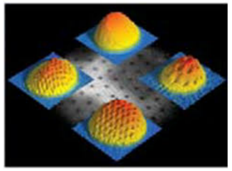
diatomic molecules

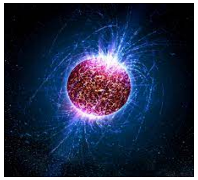


strongly interacting pairs



Cooper pairs





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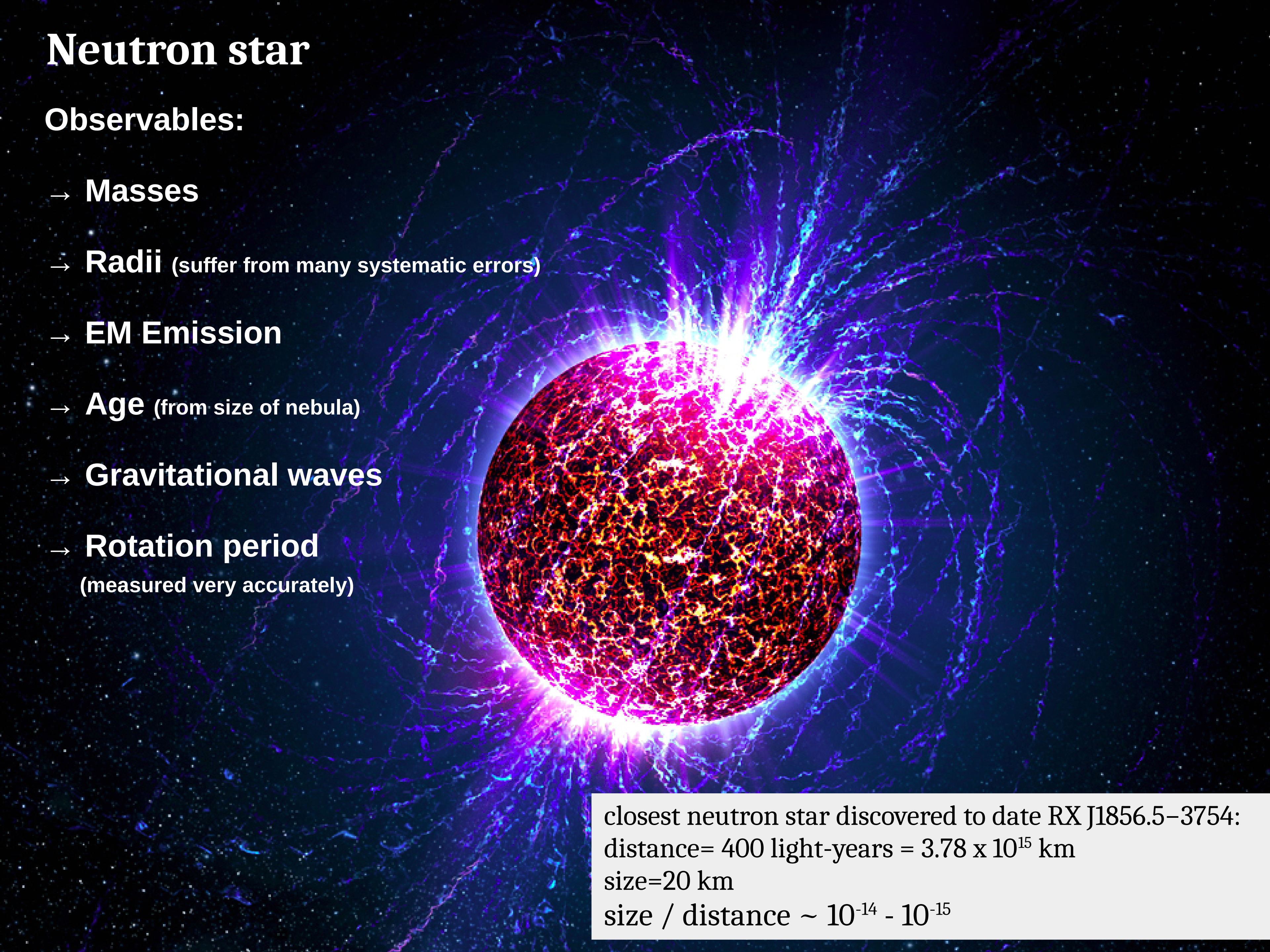
NATIONAL SCIENCE CENTRE
POLAND

XVII Polish Workshop on Relativistic Heavy-Ion Collisions
Warsaw University of Technology, December 14-15, 2024

Neutron star



Observables:

→ **Masses**

→ **Radii** (suffer from many systematic errors)

→ **EM Emission**

→ **Age** (from size of nebula)

→ **Gravitational waves**

→ **Rotation period**

(measured very accurately)

closest neutron star discovered to date RX J1856.5−3754:
distance= 400 light-years = 3.78 x 10^{15} km
size=20 km
size / distance ~ 10^{-14} - 10^{-15}

Neutron star

Observables:

→ **Masses**

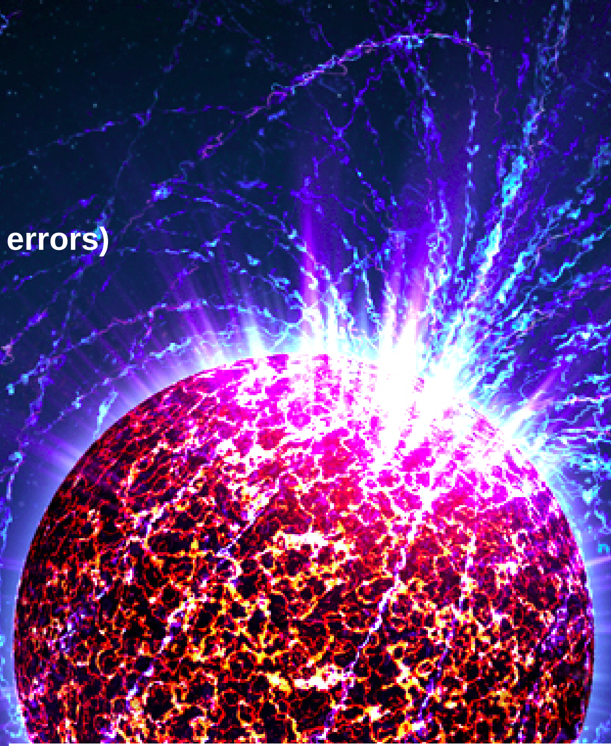
→ **Radii** (suffer from many systematic errors)

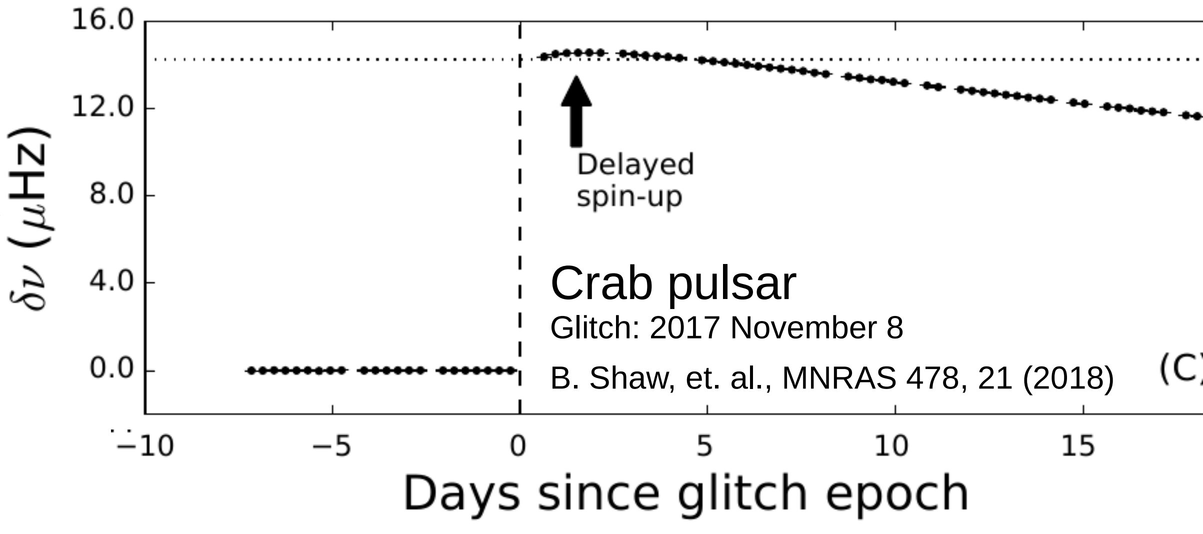
→ **EM Emission**

→ **Age** (from size of nebula)

→ **Gravitational waves**

→ **Rotation period**
(measured very accurately)





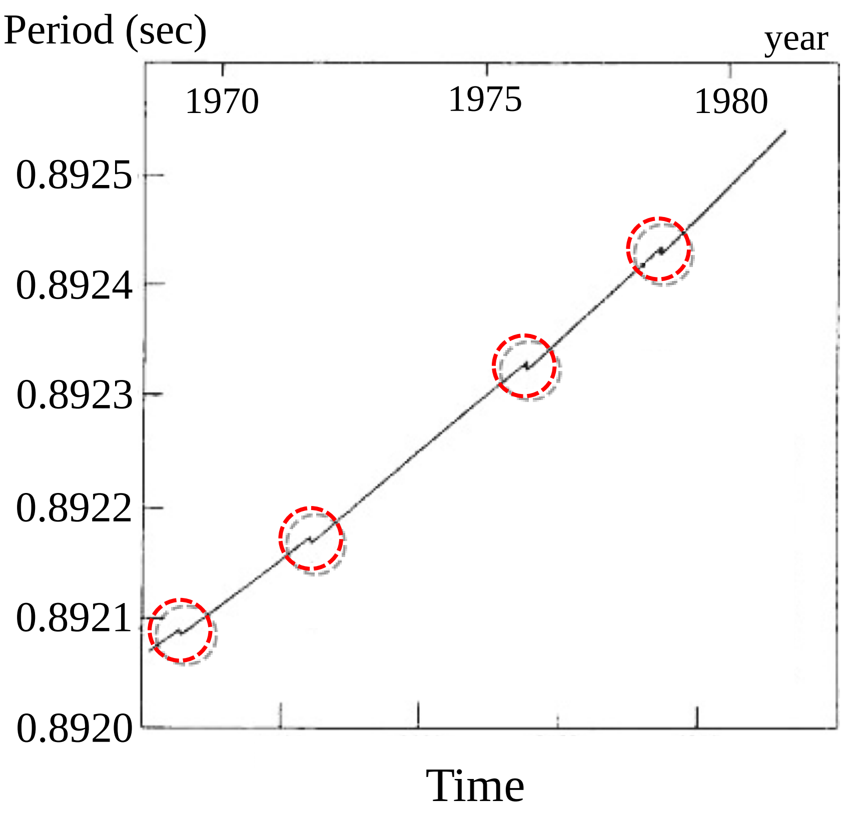
Crab pulsar

Glitch: 2017 November 8

B. Shaw, et. al., MNRAS 478, 21 (2018)

Glitch: a sudden increase of the rotational frequency

- Glitches in the Vela pulsar



V.B. Bhatia, A Textbook of Astronomy and Astrophysics
with Elements of Cosmology, Alpha Science, 2001.

➢ Glitches due to quantum vortices

(P. W. Anderson and N. Itoh, Nature 256 (1975))

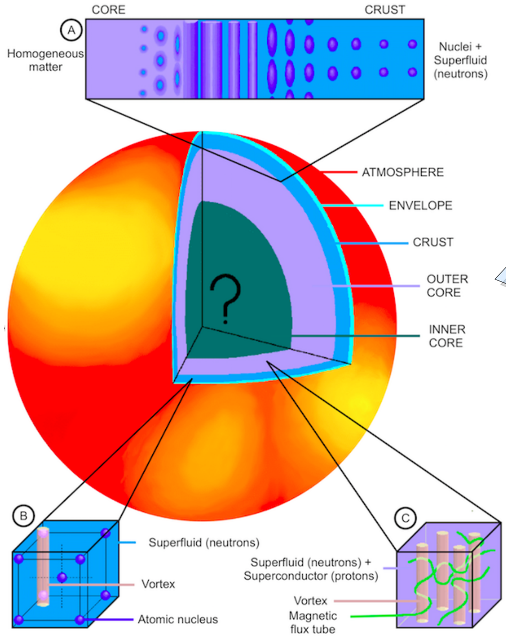
- Presently the standard picture for pulsar glitches
- Can explain: post-glitch relaxation, statistics of the glitching populations...
- Idea:
  - Superfluid interior contains quantized vortices pinned to the crustal lattice
  - Glitches are believed to occur when a large number of vortices simultaneously unpin and move outward

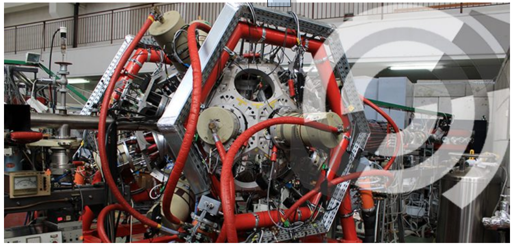
Simulation of: Vortex Avalanches and Collective Motion in Neutron Stars, I-Kang Liu, Andrew W. Baggaley, Carlo F. Barenghi, Toby S. Wood, arXiv:2410.16878

First observed in 1969: V. Radhakrishnan and R. N. Manchester, Nature 222, 228–229 (1969);
P. E. Reichley and G. S. Downs, Nature 222, 229–230 (1969);

Density Functional Theory (DFT): Workhorse for ...

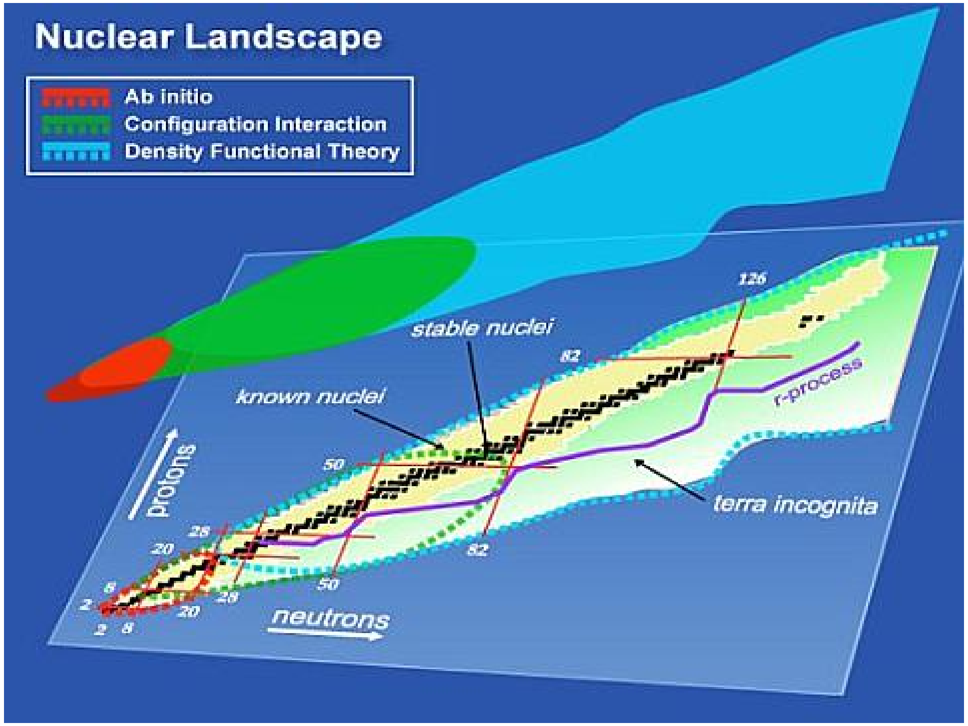
- Solid-state physics
- Quantum chemistry
- Condensed-matter physics





… also important tool for

- Nuclear physics
- (Nuclear) astrophysics
- … plasma physics ...




NEWS FEATURE

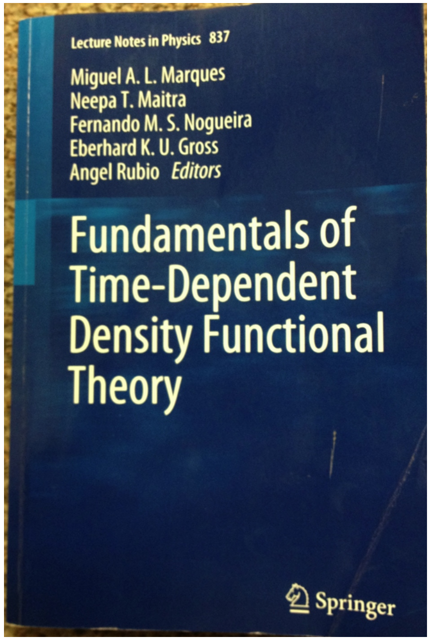
THE TOP 100 PAPERS

Nature explores the most-cited research of all time.

BY RICHARD VAN NOORDEN, BRENDAN MAHER AND REGINA NUZZO

Nature 514, 550 (2014)

… *Twelve papers on the top-100 list relate to it [DFT], including 2 of the top 10.*

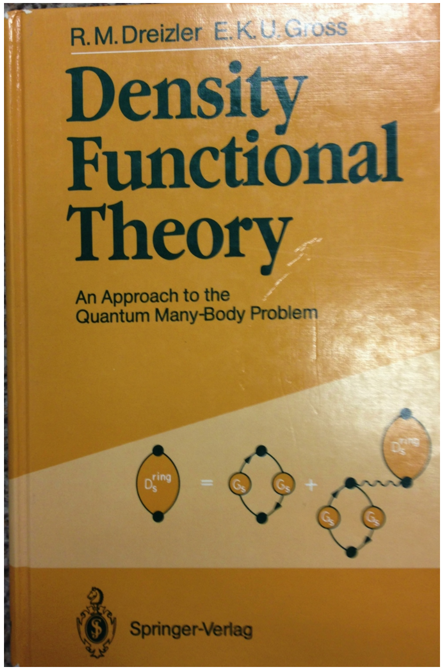


Lecture Notes in Physics 837

Miguel A. L. Marques
Neepa T. Maitra
Fernando M. S. Nogueira
Eberhard K. U. Gross
Angel Rubio *Editors*

Fundamentals of Time-Dependent Density Functional Theory

Springer

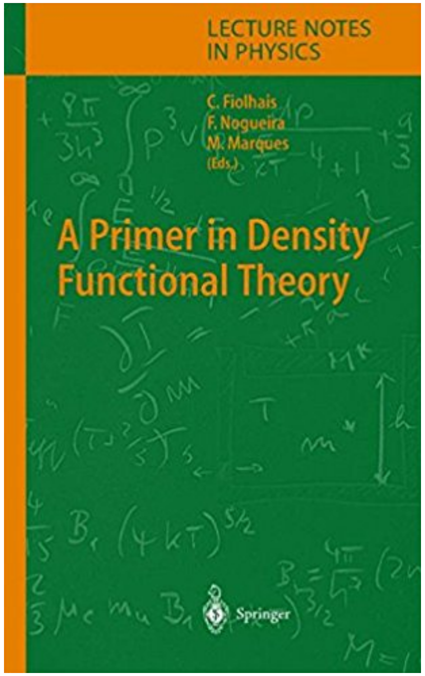


R. M. Dreizler E. K. U. Gross

Density Functional Theory

An Approach to the Quantum Many-Body Problem

Springer-Verlag



LECTURE NOTES IN PHYSICS

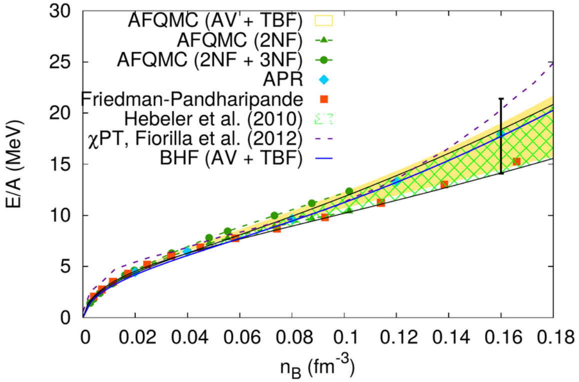
C. Fiolhais
F. Nogueira
M. Marques
(Eds.)

A Primer in Density Functional Theory

Springer

- DFT is in principle exact theory
  Hohenberg-Kohn theorem (1964) implies that $\langle O \rangle = \langle \Psi[\rho] | \hat{O} | \Psi[\rho] \rangle = O[\rho]$
- ... solving Schrödinger equation ↔ minimization of the energy density $E[\rho]$...
- ... however no mathematical recipe how to construct $E[\rho]$.
- In practice we postulate the functional form
  dimensional arguments, renormalizability, Galilean invariance, and symmetries
- Many extensions: time-dependent formalism, finite temperature,
  normal/superconducting systems, non-relativistic/relativistic, ...

EoS (typically from QMC)



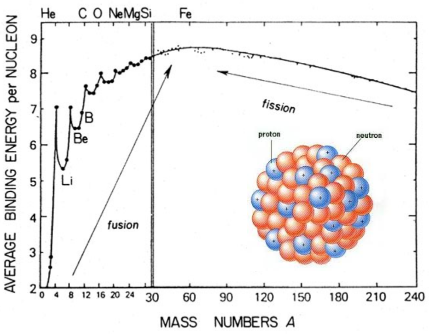
Dimensional arguments, renormalizability, Galilean invariance, and symmetries (translational, rotational, gauge, parity, ...)

Postulate

Validation against other quantities

Input 1

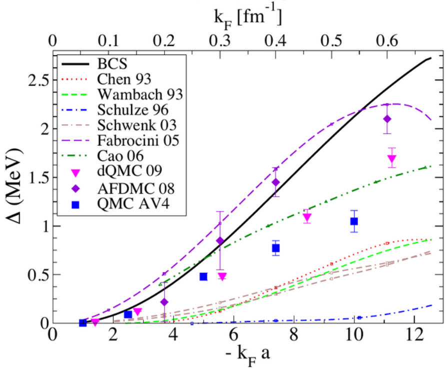
Exp. data for nuclei
(masses, radii, ...)



Input 2

Energy Density Functional E[n,...]

Pairing gap (s-wave)



Input 3

Input N

....

Predictions...

The quality of DFT results strongly depends on the quality of the energy functional, which in turn depends on the quality of the input data.

Brussels Skyrme functionals BSk(G)

We have fitted a series of nuclear energy-density functionals with full HFB calculations using extended Skyrme functionals

Experimental data/constraints:

- $\sim$ 2300 atomic masses (rms $\sim 0.5 - 0.6$ MeV$/c^2$)
- $\sim$ 900 nuclear charge radii (rms $\sim$ 0.03 fm)
- symmetry energy $29 \leq J \leq 32$ MeV
- incompressibility $K_v = 240 \pm 10$ MeV (giant resonances in nuclei)

Many-body ab initio calculations:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter (+giant resonances in nuclei)
- stability against spin and spin-isospin fluctuations

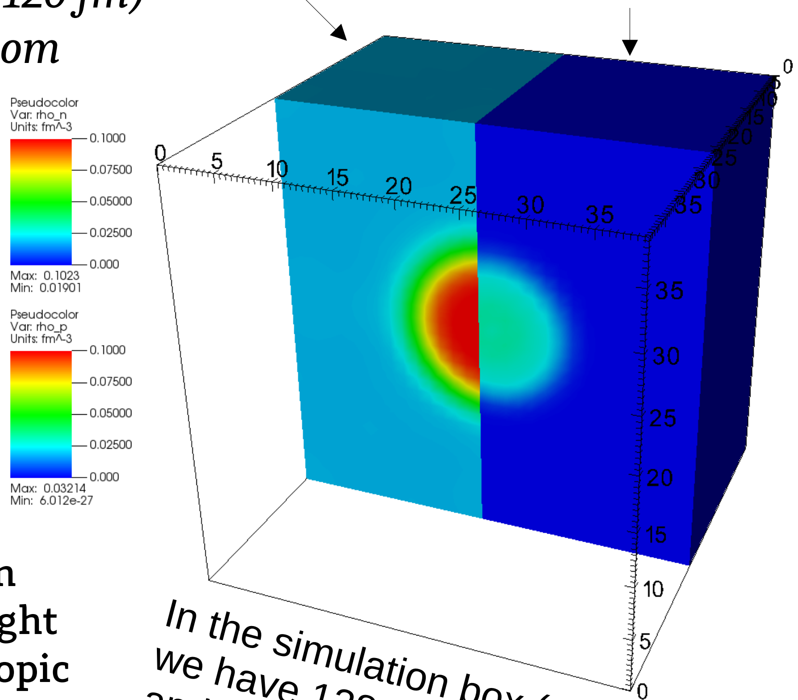
Grams et al., Eur. Phys. J. A 59, 270 (2023)

Slide from Nicolas Chamel's talk, ECT* Workshops, Trento, Apr. 2024

Today's capabilities of TDDFT (with nuclear functionals)

In context of nuclear applications

- *Unconstrained dynamics in 3D, volumes reaching V=(120 fm)^3*
- *Protons and neutrons as (dynamical) degrees of freedom*
- *Systems consisting of tens of thousands of particles*



In the simulation box (40 fm)^3 we have 1382 neutrons and 40 protons.

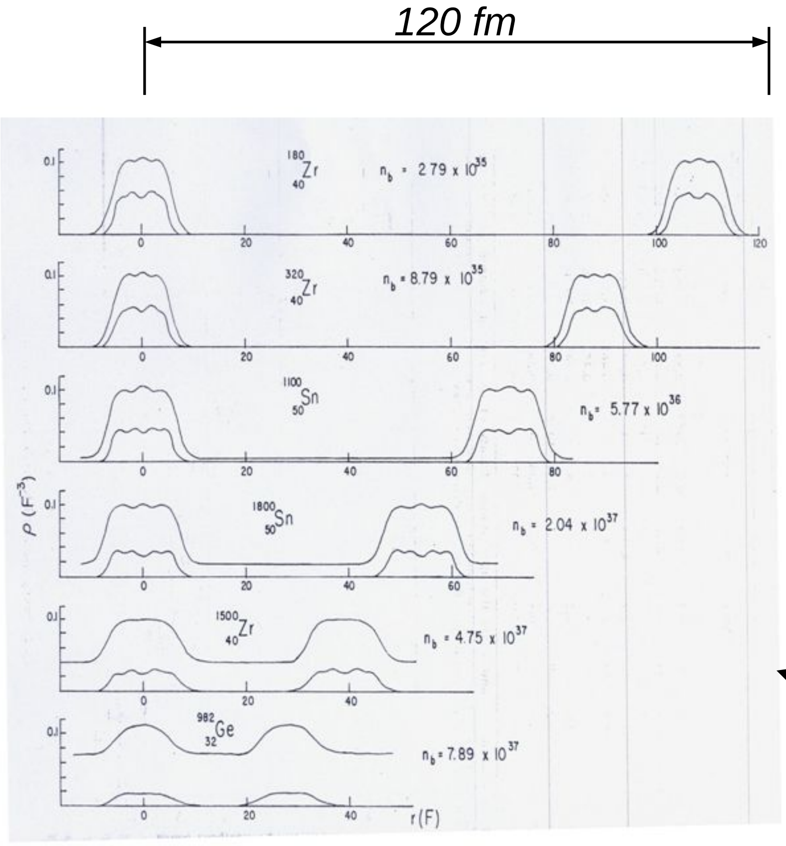
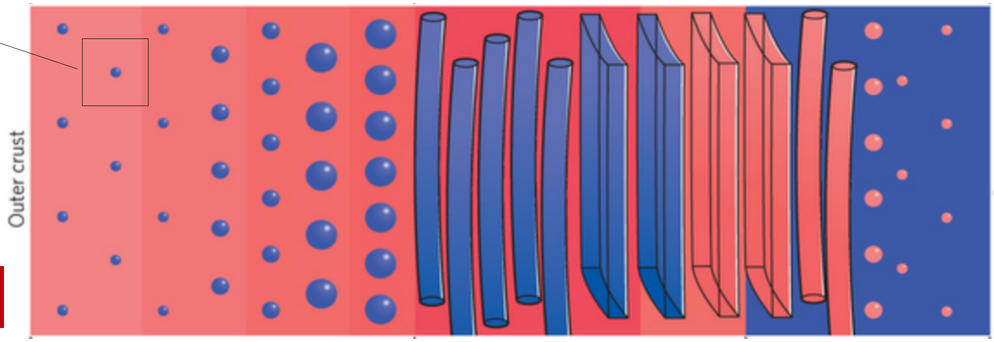


Fig. from: J.W. Negele, D. Vautherin, NPA207 (1973) 298

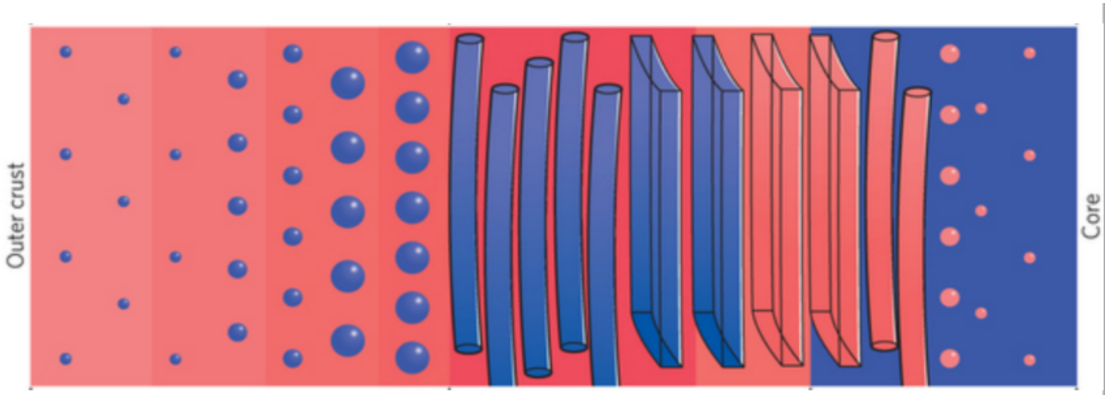
TDDFT can provide insight into microscopic dynamics on scales of Wigner-Seitz cell scale.

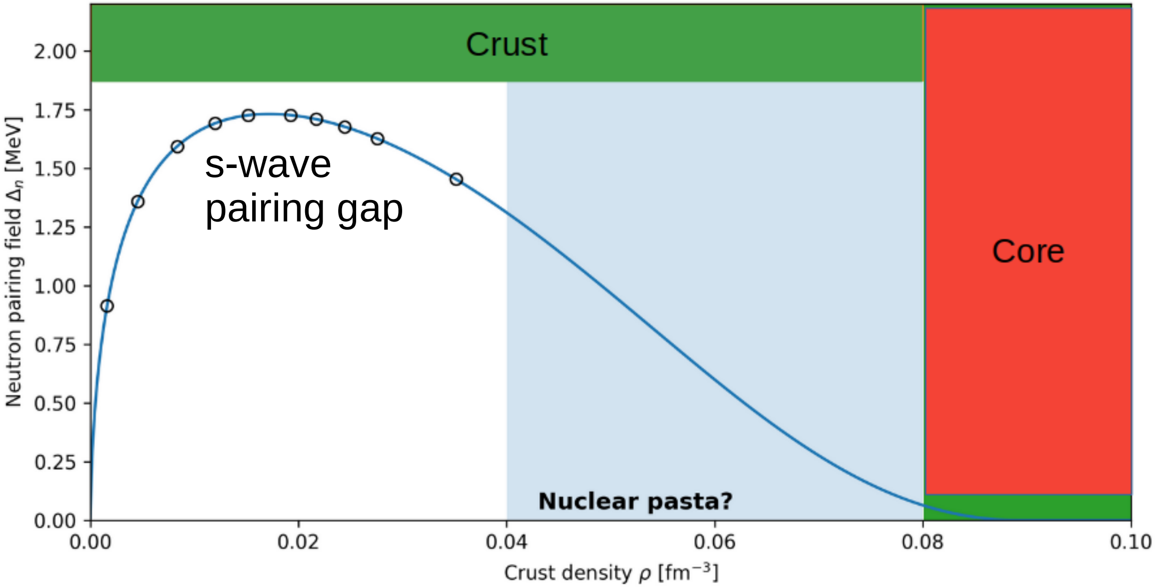


Limitations of the presented framework

- *Non-relativistic description* *[$v_F(0.08 fm^{-3})$=28% of c]*
- *S-wave superfluidity*

Neutron star crust





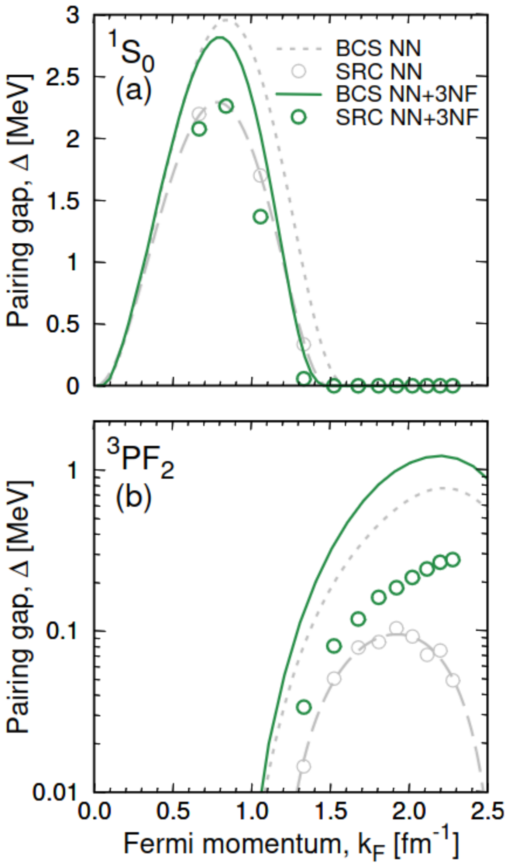
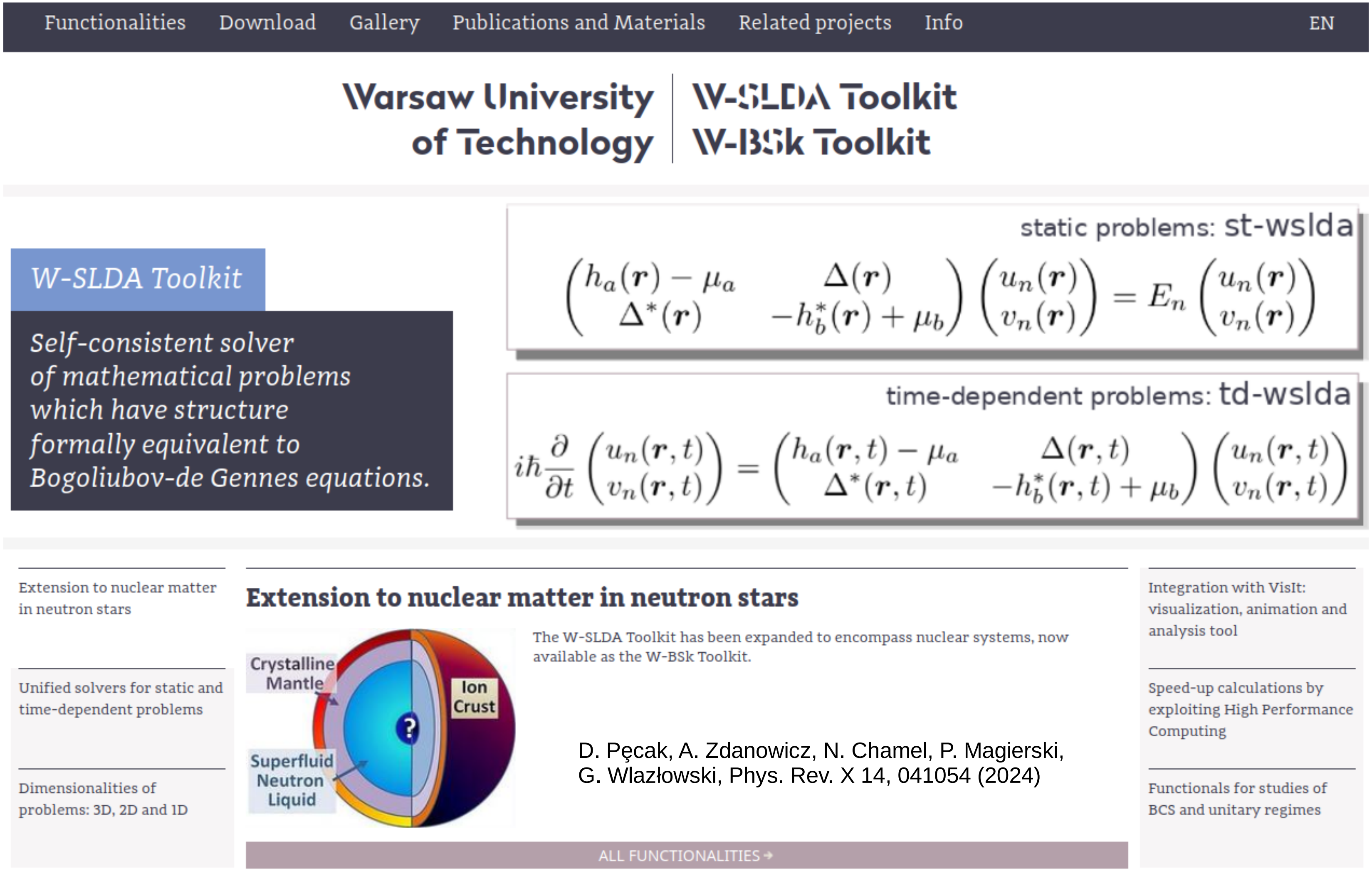


Fig from:
D. Ding, A. Rios, H. Dussan, W. H. Dickhoff, S. J. Witte, A. Carbone, and A. Polls, Phys. Rev. C 94, 025802 (2016)



Warsaw University of Technology | W-SLDA Toolkit
W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

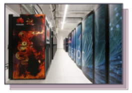
- Extension to nuclear matter in neutron stars
- Unified solvers for static and time-dependent problems
- Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars

The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

D. Pęcak, A. Zdanowicz, N. Chamel, P. Magierski, G. Wlazłowski, Phys. Rev. X 14, 041054 (2024)

- Integration with VisIt: visualization, animation and analysis tool
- Speed-up calculations by exploiting High Performance Computing
- Functionals for studies of BCS and unitary regimes



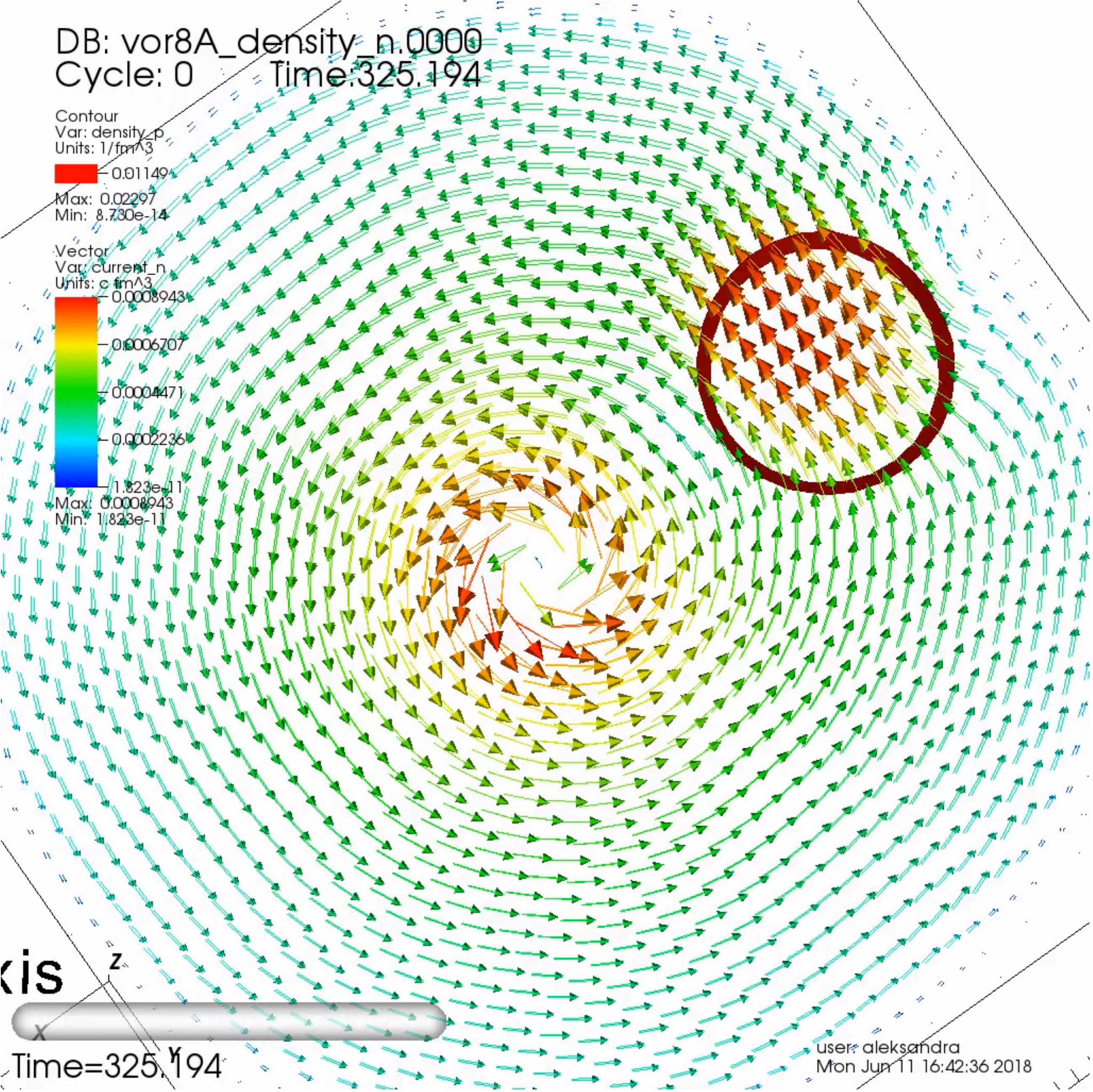
can run on “small” computing clusters as well as leadership supercomputers
(depending on the problem size)

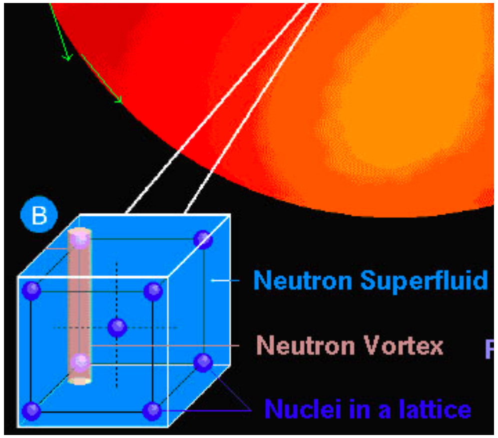
High
Performance
Computing

NVIDIA CUDA AMD ROCm open source

http://wslda.fizyka.pw.edu.pl/

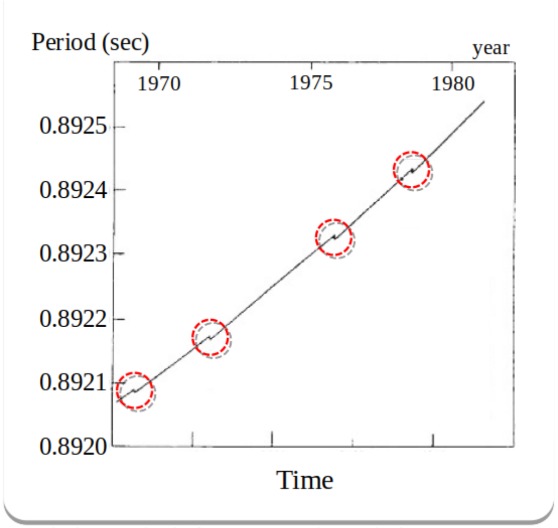
PHYSICS.WUT



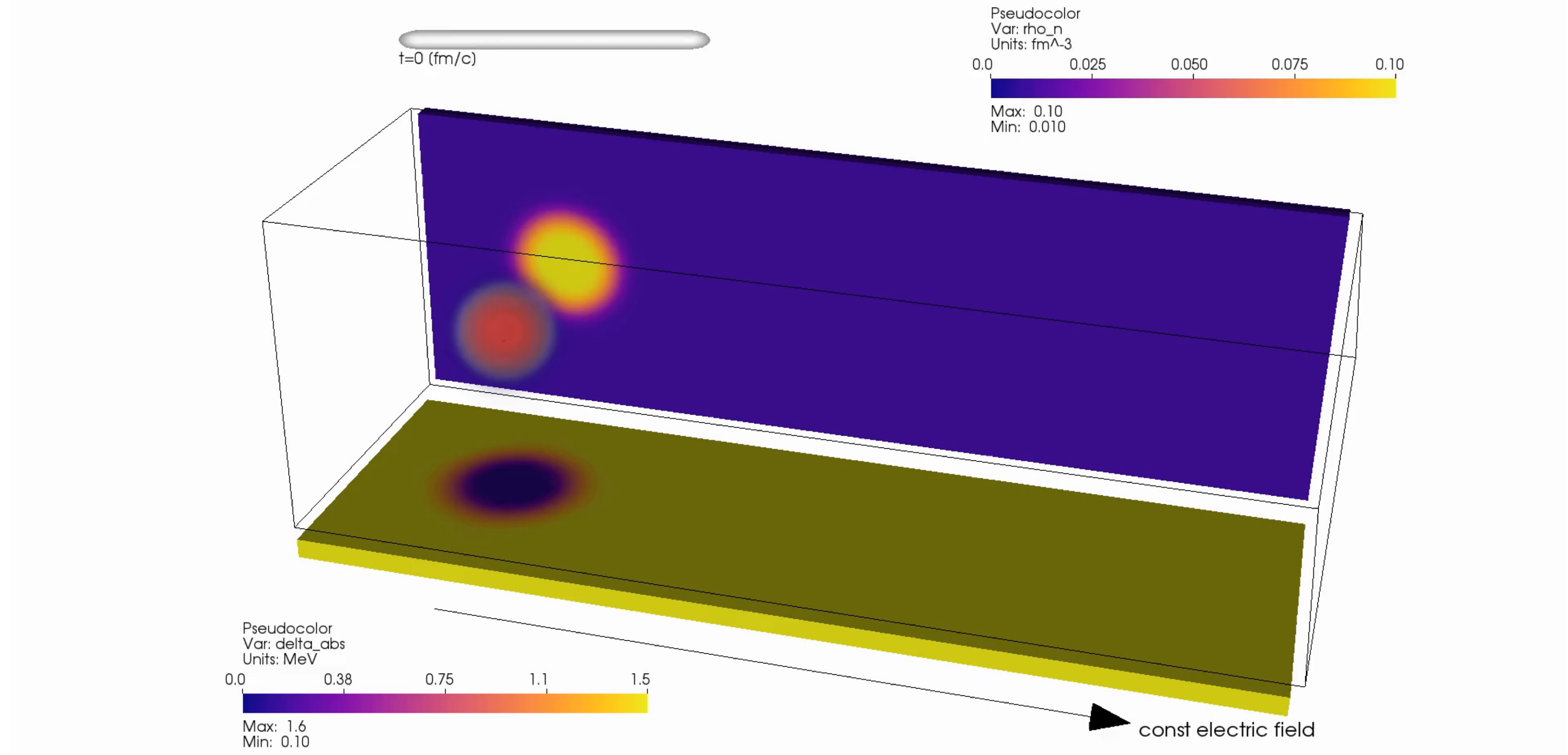


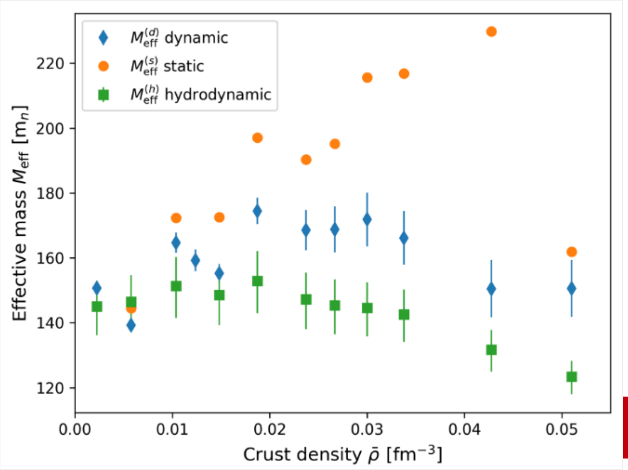
System: *nuclear in presence of quantum vortex*

Understanding of the vortex–impurity interaction is required in order or understand the phenomenon of neutron star glitches.

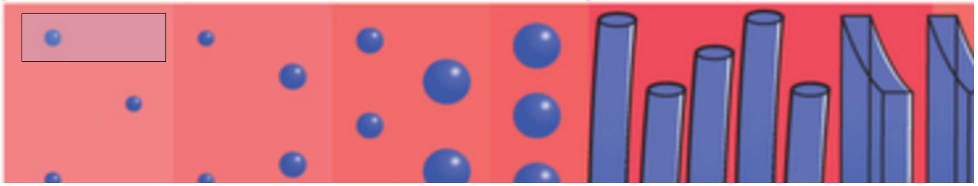


V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.





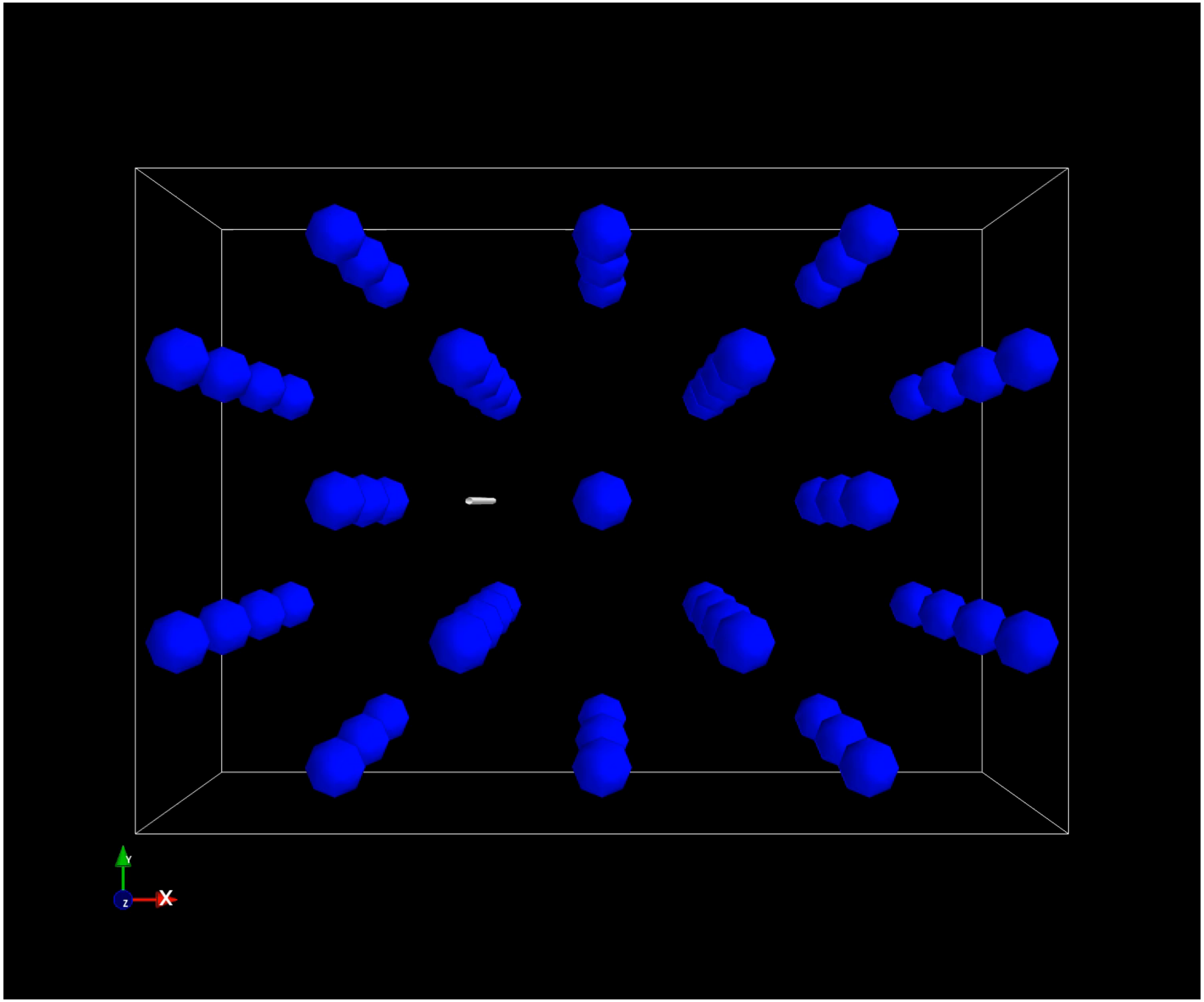
System: *nuclear matter,* 3D simulation 40 x 40 x 120 [fm]
number of neutrons: 2,104; number of protons: 40



response of nuclear impurity to uniform electric field

Phys. Rev. X 14, 041054 (2024)

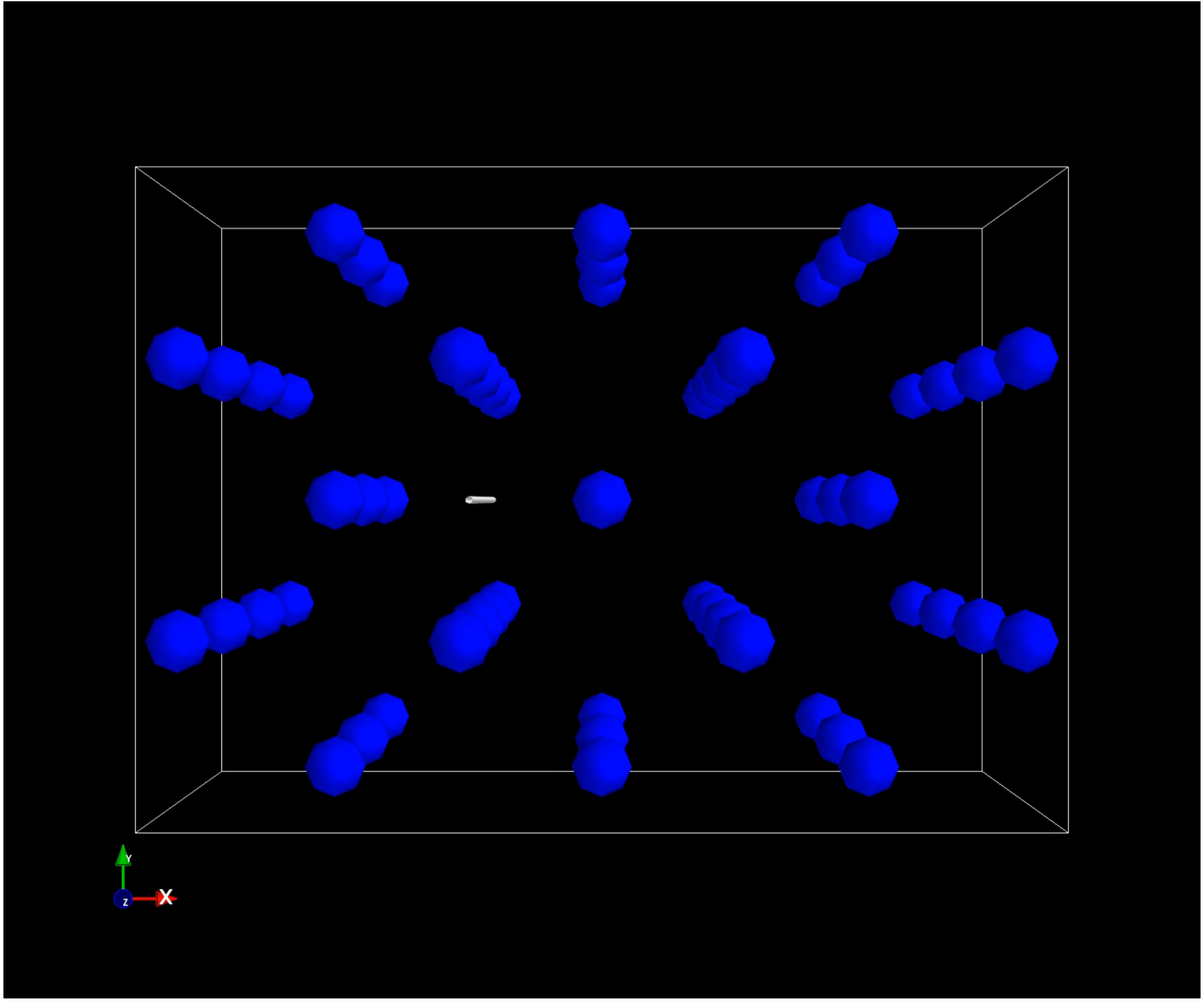
Providing microscopic inputs for mesoscopic models...



$\longrightarrow$ $V_{ext} < v_{crit}$

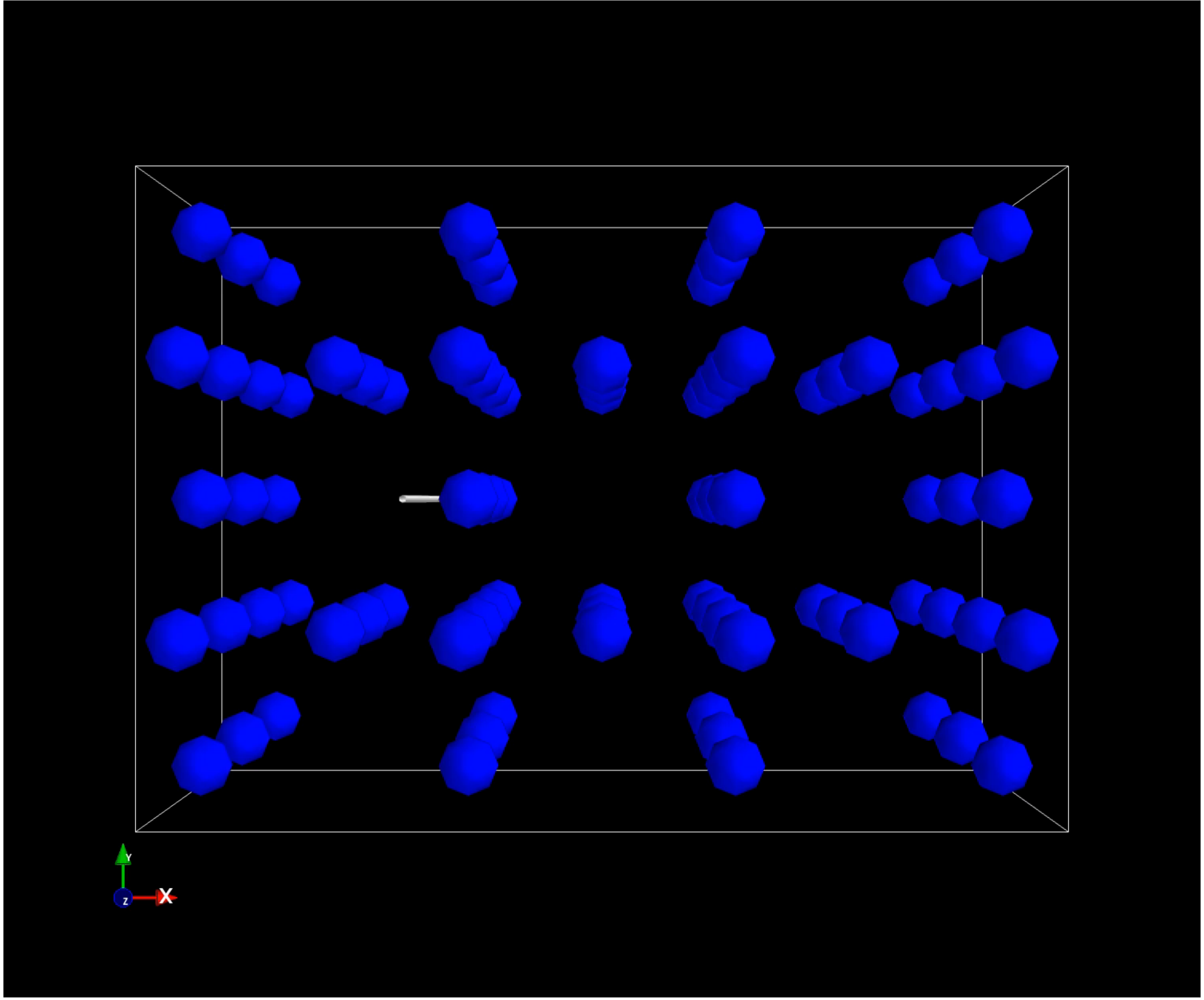
From diploma thesis of Konrad Kobuszewski, WUT 2018

Providing microscopic inputs for mesoscopic models...



$\longrightarrow$ $V_{ext} > v_{crit}$

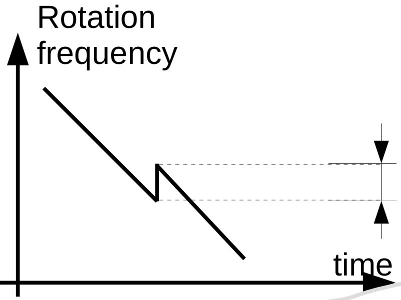
Providing microscopic inputs for mesoscopic models...



→ $V_{ext} \approx v_{crit}$

From diploma thesis of Konrad Kobuszewski, WUT 2018

Getting knowledge about the core by constraining the crust...



maximum glitch amplitude

Observed relative sizes of glitches:

$\Delta\nu/\nu \sim 10^{-11} - 10^{-5}$

$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa\, I}\int_0^{R_d} dr\, r^3\, f_P(r)$$

quantum of circulation

moment of inertia

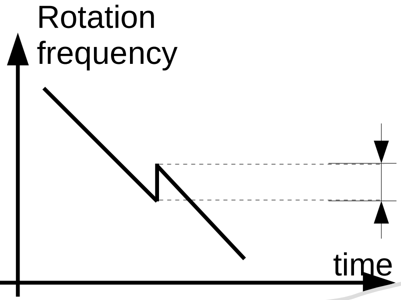
pinning force

(force needed to move a vortex through the medium with "impurities")

> Result is weakly sensitive to various assumptions of a model...

- P. Pizzochero, M. Antonelli, B. Haskell, S. Seveso, Nature Astronomy 1, 0134 (2017)
- M. Antonelli, P. Pizzochero, Journal of Physics: Conf. Series 861 (2017) 012024
- M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)

Getting knowledge about the core by constraining the crust...



maximum glitch amplitude

Observed relative sizes of glitches:

$$\Delta\nu/\nu \sim 10^{-11} - 10^{-5}$$

$$\Delta\Omega_{\mathrm{max}} = \frac{\pi^2}{\kappa\, I}\int_{R_{crust}}^{R_d} dr\, r^3\, f_P(r)$$

quantum of circulation

moment of inertia

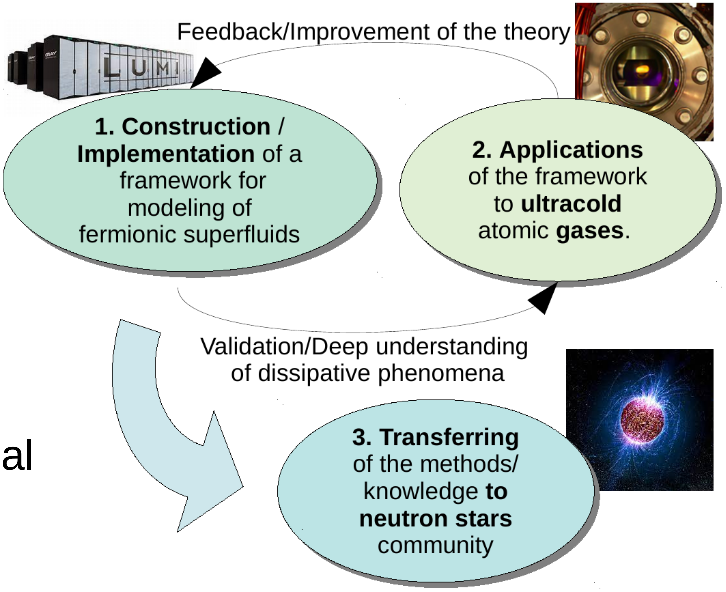
Assumption: only crust contributes (no superfluidity in the core)

> *Can we get the observed glitch sizes by assuming that only the crust is superfluid?*
> → *needed reliable (at quantitative level) calculations for the crust*

- P. Pizzochero, M. Antonelli, B. Haskell, S. Seveso, Nature Astronomy 1, 0134 (2017)
- M. Antonelli, P. Pizzochero, Journal of Physics: Conf. Series 861 (2017) 012024
- M. Antonelli, A. Montoli, P. M. Pizzochero, MNRAS 475, 5403 (2018)

SUMMARY

- Ultracold Fermi gases and neutron matter share a lot of similarities. UFG regime can be used as a benchmark platform for testing the predictive power of many-body techniques, which are subsequently used for neutron star studies.
- (TD)DFT is general purpose framework: it overcomes limitations of mean-field approach, while keeping numerical cost at the same level as (TD)HFB calculations.
- For problems that have been (so far) contrasted with experimental measurements: *Predictions by functionals for ultracold Fermi gases (SLDA), created within similar methodology as for nuclear systems, are at least at the qualitative level in agreement with the measurements, ... in many cases, good quantitative agreement is obtained.*
- (TD)DFT and its implementations reached the level of maturity that allows for providing predictions for large and complex systems: neutron star's crust structure and its dynamics, transport coefficients, …



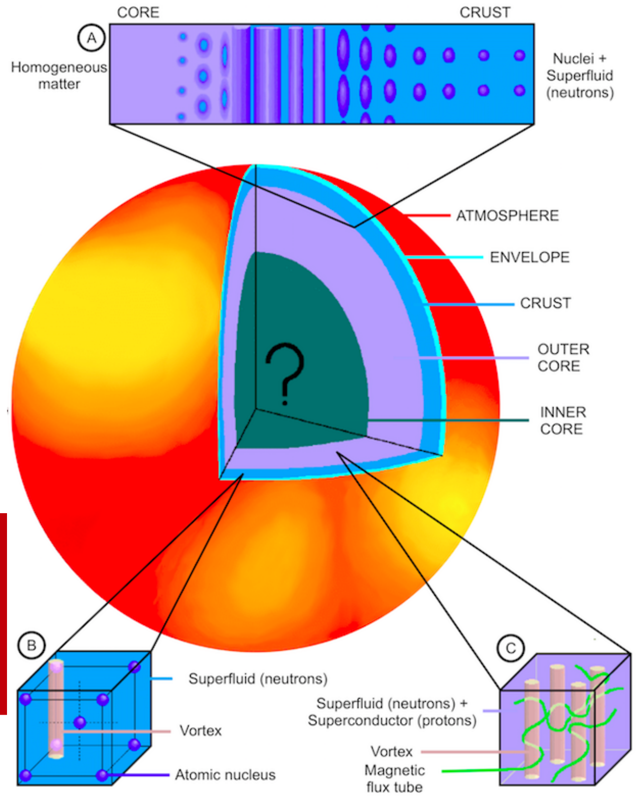
Feedback/Improvement of the theory

1. Construction / Implementation of a framework for modeling of fermionic superfluids

2. Applications of the framework to **ultracold** atomic **gases**.

Validation/Deep understanding of dissipative phenomena

3. Transferring of the methods/ knowledge **to neutron stars** community



WUT Group: P. Magierski, G. Wlazłowski, D. Pęcak, M. Tylutki,
A. Barresi, E. Alba, V. Allard, A. Zdanowicz,
M. Śliwiński, D. Lazarou; A. Makowski
In collboration with: N. Chamel (U. Bruxelles);

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http://wlazlowski.fizyka.pw.edu.pl

https://wslda.fizyka.pw.edu.pl/

NONEQUILIBRIUM PHENOMENA IN SUPERFLUID SYSTEMS: ATOMIC NUCLEI, LIQUID HELIUM, ULTRACOLD GASES, AND NEUTRON STARS

12 May 2025 — 16 May 2025

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ACKNOWLEDGMENTS

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We acknowledge Polish high-performance computing infrastructure PLGrid for awarding this project access to the LUMI supercomputer, owned by the EuroHPC Joint Undertaking, hosted by CSC (Finland) and the LUMI consortium through PLL/2022/03/016433